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PREFACE

This special issue is dedicated to the autumn school ASEM23 devoted to Equilibrium Problems and Minimax Inequalities, which was held during September 25-26, 2023 in El Jadida, Morocco, and the autumn school ASSVAS23 on Set-valued and Variational Analysis: Applications to physics and economy organized in Safi, Morocco, on two sessions: September 28-29 and November 15-16, 2023. These events are the fruit of a deep cooperation between the laboratory of fundamental mathematics and applications of El Jadida and the former Laboratory of fundamental and applied physics of Safi (LPFAS) whose name is changed now into: Laboratory of Physics, Energies, Environments and Their Applications (LP2EA). A warm acknowledgment goes to the members of these structures for their deep involvement in the organization of our workshops, especially Prof. Ahmed Serhir from LMFA laboratory, the director of LPFAS laboratory Prof. Saida bahsine, the deputy director of LPFAS Prof. Ahmed Daassou and the director of LP2EA Prof. Rachid Benbrik. It is our immense pleasure to acknowledge here the support of our partners: Chouaïb Doukkali University (UCD), Cadi Ayyad University, Moroccan National Center for Scientific and Technical Research (CNRST), the Faculty of Science of El Jadida and the Polydisciplinary faculty of Safi and the Marine Fisheries Institute of Safi. We would also like to thank all the referees who accepted to devote their precious time in a quite long and deep evaluation process.

Professor Hedy Attouch was originally scheduled to be one of the keynote speakers at these two workshops, but fate had other plans. In honor of his memory, we dedicate our humble contribution in this issue to him, hoping that his family and scientific collaborators find solace in this tribute to a remarkable scientist. His profound contributions to nonlinear analysis and optimization have left a lasting impact on the mathematical community.

This issue, which consists of two parts (Issue 3, Volume 7 and Issue 1, Volume 8) includes thirteen contributions selected among the best ones presented in the two workshops of El Jadida and Safi, which essentially cover minimax theorems and equilibrium problems, continuous dynamic systems with fast convergence, optimal control of epidemic models, renewal equations with infinite delay applied to epidemic model with waning immunity, quantitative stability of

the second order Cauchy problem of ordinary differential equations with initial-value data, extended real-valued equilibrium problem and random hemivariational inequalities, parametric time-dependent quasi-variational inequalities and elastic traffic networks, two-stage Nash equilibria with application to climate change, Generalized Nash equilibrium problems via approximate optimality of multiobjective optimization problems, power inequalities for log-concave functions, and optimality conditions for quasiconcave programming problems.

M. A. Ighachane, M. Bouchangour, and M. W. Alomari in their paper entitled 'Some generalizations of reverse power inequalities for log-concave functions' developed first new power inequalities for log-concave functions by obtaining an extension and unification of related recent results on this topic. They proved the generalized multiple term reverses refinements of the difference between the arithmetic and power means inequality for scalars and operators.

In the paper titled 'Some of the latest applications of certain minimax theorems', B. Ricceri fundamentally explored exact computation of the infimum of certain functionals on L^p spaces, the multiplicity of both global minima in a non-convex setting and periodic solutions for Lagrangian systems of relativistic oscillators. A new challenging conjecture was also proposed by the author for forthcoming perspectives intimately linked to applications of minimax Theorems.

The paper 'A new proof of the Von Neumann minimax Theorem' by M. Ait Mansour, J. Lahrache, and N. Ziane suggested a new method to prove this classic minimax Theorem by relying on standard arguments of convex optimization and Euclidean geometric elementary properties.

The paper by M. A. Diop, M. Elghandouri, and K. Ezzinbi titled 'Optimal control of general impulsive VS-EIAR epidemic models with applications to Covid-19' involves several ordinary differential equations that take into consideration the changes in the vaccinated, susceptible, infected, exposed, asymptomatic, and deceased population groups. The focus of the authors is to reduce the number of those individuals by administering vaccination doses to susceptible individuals and treatment to infected population. In doing so, the authors utilized optimal control theory to regulate the dynamic of their epidemic model within a terminal optimal time, wherein they apply Pontryagin's Maximum Principle (PMP) to find an optimal control time. Further discussions by the authors on their model take into account the factor of immigration or the travel of certain population groups. This contribution provides also a numerical simulation to support their theoretical findings.

F. Battahi, Z. Chbani, and H. Riahi, in their paper 'Tikhonov regularization to solve convex minimization with affine constraints' proposed a second-order time-continuous dynamic system with fast convergence guarantees to solve general convex minimization problems with linear constraints in Hilbert space. This work develops, in particular, a Lyapunov analysis which provides fast convergence properties of the values and of the feasibility gap in addition to presentation of numerical experiments. They concluded their results with an extension to non-smooth convex functions with extended real values.

The aim of the paper 'On random hemivariational inequalities - some solvability and stability results' by J. Gwinner is to study various classes of parameter dependent hemivariational inequalities trough mixed random hemivariational inequalities and give solvability results in the Bochner-Lebesgue space and real separable Hilbert spaces, which allows the author to focus on a probability space (Ω, P) and derive a stability result for mixed random hemivariational

inequalities together with an application to nonsmooth boundary value problem with unilateral, friction-like, and nonmonotone boundary conditions under uncertainty.

A. Afoukal, M. El Attaouy, and K. Ezzinbi, in the paper 'Stepanov oscillatory type solutions for renewal equations with infinite delay: application to an epidemic model with waning immunity' proposed a new result on the reduction principle for nonhomogeneous autonomous linear renewal equations. This result, combined with the classic formula of variation of constants, allows the authors to obtain a Stepanov's version of Massera and Bohr-Neugebauer-type results. Furthermore, they discussed uniqueness conditions and support their theoretical results by an application to an epidemic model with waning immunity.

Our issue contains a third work in connection with differential equations but rather from a quantitative stability viewpoint. Precisely, in the paper 'On the stability of second order parametric ordinary differential equations and applications' Z. Mazgouri and A. El Ayoubi devoted their efforts to Lipschitz stability for a parametric version of the general second order Ordinary Differential Equation (ODE) initial-value Cauchy problem. Indeed, they first obtained quantitative stability of a Lipschitzian type by means of a direct computation relying on Perov's inequality. Then, they derived a similar stability result for second order differential equations governed by cocoercive operators. Some concrete applications of the stability for two specific applied mathematical models inherent in electricity and control theory were also discussed together with numerical tests based on the software source Scilab.

A further contribution on variational inequality problems but in the more delicate quasi-variational format that takes into account the dependence on time at each moment beyond the presence of different perturbing parameters. These considerations lead to time-dependent parametric quasi-variational inequalities, which were presented in the paper by M. Ait Mansour and A. Barbagallo under the title 'Stability of time-dependent elastic Cournot-Nash equilibrium in oligopolistic markets under perturbation'. As a first step of this work, the authors presented an abstract quantitative stability result, by establishing estimates with sharp error bounds, on general mathematical time-dependent quasi-variational problem by involving suitable conditions. The suggested assumptions arrange an efficient combination with stability properties on the underlying fixed points. In the second step of their contribution, they applied the stability of their main result to parametric market equilibrium problem supported by a numerical illustration.

The issue continues the exploration on Nash equilibrium problems following two further different ways. One of them concerns a two stage format with a focus on a modern and challenging application to the climate change, which makes the object of the paper by M. Passacantando and F. Raciti titled 'Adapting to climate change: a two-stage Nash equilibrium model of coalition formation.' In this work, the authors considered a two-stage noncooperative game problem to formulate the coalition formation of countries which decide to jointly invest in research and developments projects to mitigate the damages induced by climate changes. A key step in this treatment relies on an equivalent reformulation of the variational equilibrium of the second-stage game as a variational inequality with a reduced number of variables and monotonicity conditions. A numerical investigation on the impact of the coalition on the reduction of the environmental damage closes this contribution by Passacantando and Raciti.

Another interesting generalized Nash equilibrium problem in a multiobjective form is the aim of the paper by R. EL. Idrissi, E. M. Kalmoun, and L. Lafhim in their paper 'New approximate optimality conditions for strong type in multiobjective generalized Nash equilibrium

problem' (MGNEPs for short), wherein after that the authors observed that any efficient solution satisfies the standard approximate strong Karush-Kuhn-Tucker (KKT) condition, they recognized that these conditions may be overly stringent. Then, they proposed a new approximate strong KKT condition specifically tailored for MGNEPs, named MGNEP-ASKKT. The convergence of a MGNEP-ASKKT sequence to a MGNEP-SKKT point in this work was ensured by introducing the concept of cone-continuity regularity. Moreover, the authors provided an enhanced Lagrangian-type algorithm for solving MGNEPs together with its global convergence to a MGNEP-SKKT point.

A. Hassouni, M. Berdi, and M. A. Abouchouar in the paper titled 'Sufficient optimality conditions for a separable product quasiconcave programming' studied sufficient optimality conditions for a multiplicatively separable non-differentiable quasiconcave programming in both unconstrained and inequality constrained cases. In this way, the authors highlighted the crucial role of quasiconcavity property in the optimization field by stressing the use of multiplicative separability conditions.

For a minimax approach in the context of a recent uniqueness problem, a global minimum for appropriate functions, posed by B. Ricceri under the motivation of establishing new fixed point assertions in non usual ways. This contribution, under the title 'Some fixed point results via a minimax approach' by M. Ait Mansour, J. Lahrache, and N. Ziane, brings several examples in finite dimensional spaces and stimulate further efforts for forthcoming works on the general challenging infinite dimensional case.

Lastly, we extend our sincere gratitude to everyone who contributed to and supported this special issue. We are especially thankful to all the authors for their valuable contributions and to the reviewers for their time, expertise, and constructive feedback. We also deeply appreciate the Editors-in-Chief of Applied Set-Valued Analysis and Optimization for their unwavering support and for granting us the opportunity to organize this special issue.



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