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ON SPLIT NULL POINT AND FIXED POINT PROBLEMS FOR DEMIMETRIC MAPPINGS

OLUWATOSIN TEMITOPE MEWOMO[1,*], ABD-SEMII OLUWATOSIN-ENITAN OWOLABI[2], TIMILEHIN OPEYEMI ALAKOYA[3], LANRE AKINYEMI[4]

[1]*Department of Mathematics, Tarleton State University, Stephenville, Texas, United States*
[2]*Department of Mathematics and Statistics, Kampala International University, Kampala, Uganda*
[3]*Mathematical Sciences Research Centre, School of Mathematics and Physics,*
Queen's University Belfast, Belfast, United Kingdom
[4]*Department of Mathematics, Prairie View A& M University, Texas, United States*

Abstract. The Split Null Point Problem (SNPP) was extensively studied due to its wide applications and various iteration methods were proposed for its solution. However, most of the existing methods in the literature require prior knowledge of the operator norm, which in most cases is very difficult or almost impossible to compute or even estimate. In this paper, we study the SNPP of maximal monotone mappings and fixed point problems of demimetric mappings. We introduce a self-adaptive step size algorithm of inertial viscosity method for finding a common solution of the problems. The self-adaptive step size ensures no requirement for a prior knowledge or estimate of the norm of the operator. Under mild conditions, we obtain strong convergence of the proposed algorithm. Finally, we present some applications and numerical examples to demonstrate the performance of our method and compare with some related methods. Our results extend and improve several related results in the literature.
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1. INTRODUCTION

Let H_1 and H_2 be real Hilbert spaces with inner product $\langle \cdot, \cdot \rangle$ and induced norms $\| \cdot \|$. Let I be the identity operator on a Hilbert space. We denote the set of positive integers and the set of real numbers by $\mathbb{N}^*$ and $\mathbb{R}$, respectively. The Split Inverse Problem (SIP) is to find

$$\bar{x} \in H_1 \text{ that solves } IP_1 \text{ such that } A\bar{x} \in H_2 \text{ solves } IP_2,$$

where $A : H_1 \to H_2$ is a linear and bounded mapping, IP_1 and IP_2 are inverse problems in H_1 and H_2, respectively. The SIP has attracted attention from numerous researchers; see, e.g., [5, 7, 10, 26, 27] for various applications.

Let C and Q be nonempty, convex, and closed subsets of H_1 and H_2, respectively. Let $A : H_1 \to H_2$ be a linear and bounded mapping with adjoint operator A^*. The Split Convex Feasibility Problem (SCFP) is to find a point $x^* \in H_1$ such that $x^* \in C$ and $Ax^* \in Q$. The SCFP

*Corresponding author.
E-mail address: omewomo@tarleton.edu (O.T. Mewomo).
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was introduced by Censor and Elfving [7] in finite dimensional Euclidean spaces for modeling inverse problems which arise from phase retrievals, medical image reconstruction, radiation therapy, computerized tomography, and so on. Other split problems include Split Equilibrium Problem (SEP), Split Equality Feasibility Problem (SEFP), Split Common Fixed Point Problem (SCFPP), Split Monotone Variational Inequality Problem (SMVIP), and Split Equality Fixed Point Problem (SEFPP); see, [6, 18, 36, 37].

Let H_1, H_2, and H_3 be real Hilbert spaces. Let $T : H_1 \to H_1$ and $U : H_2 \to H_2$ be two nonlinear mappings, and let $A : H_1 \to H_3$ and $B : H_2 \to H_3$ be linear and bounded mapping. Recall that the SEFPP is to find $x^* \in F(T)$ and $y^* \in F(U)$ such that $Ax^* = By^*$, where $F(T)$ and $F(U)$ are sets of fixed points of T and U, respectively.

In [20], Moudafi and Al-Shemas proposed the following iterative algorithm for solving the SEFPP

$$\begin{cases} x_{n+1} = T(x_n - \gamma_n A^*(Ax_n - By_n)), \\ y_{n+1} = U(y_n + \gamma_n B^*(Ax_n - By_n)), \quad n \geq 1, \end{cases}$$

where T and U are firmly quasi-nonexpansive mappings, $\gamma_n \in \left(\varepsilon, \frac{2}{\lambda_A + \lambda_B} - \varepsilon\right)$, λ_A and λ_B are the spectral radii of A^*A and B^*B, respectively. Under appropriate conditions, weak convergence of their algorithm was obtained. However, a prior knowledge of the operator norms is required for the implementation of the algorithm.

Let C be a nonempty, convex and closed subset of a real Hilbert space H. The Equilibrium Problem (EP), introduced by Blum and Oettli [3] in 1994, is to find a point $x \in C$ such that $F(x,y) \geq 0$ for all y in set C. We denote the solution set of the EP by $EP(F)$. In view of its wide range of applications in a large variety of problems / fields, several iterative algorithms were proposed and studied for solving the EP and its variants; see, e.g., [1, 11, 14, 28] and the references therein.

Let $F_1 : C \times C \to \mathbb{R}$ and $F_2 : Q \times Q \to \mathbb{R}$ be nonlinear bifunctions. The Split Equilibrium Problem (SEP) introduced by Kazmi and Rizvi [15] is to find $\bar{x} \in C$ such that $F_1(\bar{x}, x) \geq 0$, for all $x \in C$, and $\bar{y} = A\bar{x} \in Q$ solves $F_2(\bar{y}, y) \geq 0$, for all $y \in Q$.

Recall that the Split Monotone Variational Inclusion Problem (SMVIP), which was introduced by Moudafi [21], is to find $x^* \in H_1$ such that $0 \in g(x^*) + B_1(x^*)$, and $y^* = Ax^* \in H_2$ solves $0 \in h(y^*) + B_2(y^*)$, where 0 is the zero vector, g and h are given single-valued operators, $B_1 : H_1 \to 2^{H_1}$ and $B_2 : H_2 \to 2^{H_2}$ are multivalued maximal monotone mappings, and $A : H_1 \to H_2$ is a linear and bounded mapping. If $g \equiv 0$ and $h \equiv 0$, then the SMVIP reduces to Split Variational Inclusion Problem (SVIP), which is to find $x^* \in H_1$ such that $0 \in B_1(x^*)$ and $y^* = Ax^* \in H_2$ solves $0 \in B_2(y^*)$. We denote the solution set of the SVIP by Ω, that is,

$$\Omega = \left\{x^* \in H_1 : 0 \in B_1(x^*) \text{ and } 0 \in B_2(y^*),\ y^* = Ax^*\right\}.$$

The SVIP is also known as the Split Null Point Problem (SNPP).

The following two algorithms were proposed and studied by Byrne et al. [5] for solving the SNPP for two maximal monotone operators B_1 and B_2

$$x_{n+1} = J_\lambda^{B_1}(x_n - \gamma A^*(I - J_\lambda^{B_2})Ax_n),\ n \in \mathbb{N} \tag{1.1}$$

and

$$v \in H_1,\ x_{n+1} = \alpha_n v + (1-\alpha_n) J_\lambda^{B_1}(x_n - \gamma A^*(I - J_\lambda^{B_2})Ax_n),\ n \in \mathbb{N}, \tag{1.2}$$

where $J_\lambda^{B_1}$ and $J_\lambda^{B_2}$ are resolvents of B_1 and B_2, respectively, and A^* is the adjoint operator of A. Weak convergence of iteration (1.1) was obtained while strong convergence result was obtained for iteration (1.2) under appropriate conditions.

In [32], Takahashi, Xu, and Yao studied the following problem of finding the common solution of null point and fixed point problems: Find $z \in H_1$ such that $z \in F(S) \cap B^{-1}0$ and $Az \in F(T)$, where $B^{-1}0 := \{x \in H : 0 \in Bx\}$ is the null point set of B. They introduced the following iterative algorithm

$$x_{n+1} = \alpha_n x_n + (1-\alpha_n)S(J_{\gamma_n}^B(I-\gamma_n A^*(I-T)A)x_n),\ n \in \mathbb{N},$$

where $0 < c \leq \alpha_n \leq d < 1$, $0 < a \leq \gamma_n \leq b < \frac{1}{\|A\|^2}$, $B : H_1 \to 2^{H_1}$ is a maximal monotone operator, $S : C \to C$ is a generalized hybrid operator, $T : H_2 \to H_2$ is a nonexpansive operator, $A : H_1 \to H_2$ is a linear and bounded operator and A^* is the adjoint operator of A. Under some suitable conditions, weak convergence was obtained.

In [13], Jailoka and Suantai introduced the following iterative algorithm for finding a solution of split common fixed point and null point problems for demicontractive operators and maximal monotone operators, and they obtained strong convergence of the algorithm

$$\begin{cases} y_n = x_n + \gamma A^*(S_{2[r_2]}J_{\lambda_n}^{B_2} - I)Ax_n, \\ x_{n+1} = \alpha_n f(x_n) + (1-\alpha_n)S_{1[r_1]}(J_{\lambda_n}^{B_1} y_n),\ n \in \mathbb{N}, \end{cases}$$

where $\alpha_n \in (0,1)$, $\gamma \in \left(0, \frac{2}{\|A\|^2}\right)$, $A : H_1 \to H_2$ is a linear and bounded operator, $B_1 : H_1 \to 2^{H_1}$ and $B_2 : H_2 \to 2^{H_2}$ are maximal monotone operators, $J_{\lambda_1}^{B_1}$ and $J_{\lambda_2}^{B_2}$ are the resolvents of B_1 and B_2, respectively, for $\lambda_1, \lambda_2 > 0$, $S_i : H_i \to H_i$ are k_i-demicontractive operators for $i = 1,2$, $S_{i[r_i]} = I + r_i(S_i - I)$, and $r_i \in (0, 1-k_i)$.

In [12], Jailoka and Suantai also proposed the following Halpern-type algorithm for finding a common solution of the split null point problem for maximal monotone operators and the fixed point problem of demicontractive multivalued mappings

$$\begin{cases} y_n = J_{\lambda_n}^{B_1}(x_n + \gamma A^*(J_{\lambda_n}^{B_2} - I)Ax_n), \\ u_n = (1-\sigma)y_n + \sigma z_n, \quad z_n \in Sy_n, \\ x_{n+1} = \alpha_n u + (1-\alpha_n)u_n,\ n \in \mathbb{N}, \end{cases}$$

where $\gamma \in \left(0, \frac{2}{\|A\|^2}\right)$, $\alpha_n \in (0,1)$, $\sigma \in (0, 1-k)$, $\lambda_n \in (0,\infty)$, $S : C \to CB(C)$ is a k-demicontractive multivalued mapping, $A : H_1 \to H_2$ is a linear and bounded operator, $B_1 : H_1 \to 2^{H_1}$ and $B_2 : H_2 \to 2^{H_2}$ are maximal monotone operators, $J_{\lambda_n}^{B_1}$ and $J_{\lambda_n}^{B_2}$ are the resolvents of B_1 and B_2, respectively. Strong convergence of the algorithm was obtained under some suitable conditions.

Note that the step size γ (or γ_n) of the above algorithm plays crucial role in the convergence. In addition, the algorithm requires prior knowledge of the operator norm. One of the drawbacks is that the computation of the operator norm is a very difficult task. In [40], Zhao proposed an iterative algorithm for solving the SEFPP with quasi-nonexpansive mappings, which does not require a prior knowledge of the operator norm. Under suitable conditions, weak convergence of the algorithm was obtained in real Hilbert spaces. To accelerate the rate of convergence of iterative algorithms, authors often employ the inertial technique, which was first proposed by Polyak [25] to solve the smooth convex minimization problem. The inertial step is s to use the current iteration point and the previous iteration point. Recently, researchers constructed various efficient algorithms by using inertial extrapolation (see, e.g., [22, 23, 24, 35] and the references therein).

Inspired by the results above and the current research interest in this direction, we introduce a new inertial iterative scheme with self-adaptive step sizes for finding a common solution of a split null point problem and a fixed point problem of demimetric mappings in Hilbert spaces. Our algorithm is designed such that its implementation does not require a prior estimate of the norm of the bounded linear operator. We prove strong convergence and apply our convergence result to study split minimization problems and split equilibrium problems. We present some numerical experiments to demonstrate the efficiency of the proposed algorithm in comparison with some existing results in the current literature.

This article are organised as follows: In Section 2, we recall some basic definitions and useful lemmas. In Section 3, we present our algorithm and highlight some of its important. In Section 4, we analyze the convergence of the proposed algorithm. In Section 5, we apply our result to split minimization problems and split equilibrium problems. In Section 6, we present some numerical examples to demonstrate the efficiency of our proposed algorithm in comparison with some recent results in the literature. Finally, we give the concluding remarks in Section 7.

2. PRELIMINARIES

We denote the strong convergence and weak convergence of a sequence $\{x_n\}$ to a point x in a Hilbert space H by $x_n \to x$ and $x_n \rightharpoonup x$, respectively and $w_\omega(x_n)$ denotes set of weak limits of $\{x_n\}$, that is, $\omega_w(x_n) := \{x \in H : x_{n_j} \rightharpoonup x$ for some subsequence $\{x_{n_j}\}$ of $\{x_n\}\}$. Let H be a real Hilbert space, and let C a nonempty, convex, and closed subset H. Recall that the *metric projection* $P_C : H \to C$ is defined as $||x - P_C x|| = \inf\{||x - z|| : z \in C\}$ for each $x \in H$. It is known that P_C is nonexpansive and satisfies the following properties: (1) $||P_C x - P_C y||^2 \leq \langle P_C x - P_C y, x - y\rangle$ for all $x, y \in H$; (2) for any $x \in H$ and $z \in C, z = P_C x$ if and only if $\langle x - z, z - y\rangle \geq 0$ for all $y \in C$; (3) for any $x \in H$ and $y \in C$, $||P_C x - y||^2 + ||x - P_C x||^2 \leq ||x - y||^2$; (4) for any $x, y \in H$ with $y \neq 0$, let $Q = \{z \in H : \langle y, z - x\rangle \leq 0\}$. Then, for all $u \in H$, $P_Q(u)$ is given by $P_Q(u) = u - \max\left\{0, \frac{\langle y, u-x\rangle}{||y||^2}\right\}y$, which gives an explicit formula for computing the projection of any point onto a half-space. Let H be a real Hilbert space and $\alpha \in \mathbb{R}$. Then, for all $x, y \in H$, $\|x+y\|^2 = \|x\|^2 + 2\langle x, y\rangle + \|y\|^2$; $\|x+y\|^2 \leq \|x\|^2 + 2\langle y, x+y\rangle$; and $\|\alpha x + (1-\alpha)y\|^2 = \alpha\|x\|^2 + (1-\alpha)\|y\|^2 - \alpha(1-\alpha)\|x-y\|^2$. Let C be a nonempty, convex, and closed subset of H. Recall that a mapping $f : C \to C$ is called an α-contraction if, for $\alpha \in [0, 1)$, $\|f(x) - f(y)\| \leq \alpha\|x - y\|$ for all $x, y \in C$. Let H be a real Hilbert space and $A : H \to 2^H$ be a multivalued mapping. The effective domain of A, denoted by $D(A)$ is defined as: $D(A) = \{x \in H \ : Ax \neq \emptyset\}$. The graph of A denoted by $G(A)$ is defined by $G(A) = \{(x, u) \in H \times H : u \in Ax\}$. A is monotone if $\langle x - y, u - v\rangle \geq 0$, for all $x, y \in D(A), u \in A(x)$, and $v \in A(y)$. It is maximal if the graph $G(A)$ is not contained in the graph of any other monotone operator. Clearly, A is maximal if, for any $(x, u) \in H \times H, (y, v) \in G(A)$, $\langle x - y, u - v\rangle \geq 0$ implies $u \in Ax$. The resolvent of A is defined by $J^A_\lambda := (I + \lambda A)^{-1}$, for all $\lambda > 0$. It is known that $J^A_\lambda : H \to D(A)$ is single valued and $\|J^A_\lambda x - J^A_\lambda y\|^2 \leq \langle J^A_\lambda x - J^A_\lambda y, x - y\rangle$.

Remark 2.1. For $\lambda > 0$, the following results hold [38]:

(1) B is maximal monotone if and only if J^B_λ is single-valued, firmly nonexpansive ,and $dom(J^B_\lambda) = H$, where $dom(B) := \{x \in H | B(x) \neq \emptyset\}$.

(2) The point $x^* \in B^{-1}(0)$ if and only if $x^* = J_\lambda^B x^*$.
(3) The solution set Ω of the SNPP is equivalent to the following:

$$\text{Find } x^* \in H_1 \text{ with } x^* = J_\lambda^{B_1} x^* \text{ such that } y^* = Ax^* \in H_2 \text{ and } y^* = J_\lambda^{B_2} y^*.$$

Let $S : C \to H$ be a nonlinear mapping. The mapping $I - S$ is said to be demiclosed at zero if, for any sequence $\{x_n\} \subset C$ which converges weakly to p and the sequence $\{\|x_n - Sx_n\|\}$ converges strongly to 0, $p \in F(S)$. Recall that S is said to be

(i) *k-strict pseudo-contractive* [4] if there exists a real number $k \in [0,1)$ such that

$$\|Sx - Sy\|^2 \leq \|x - y\|^2 + k\|(x - Sx) - (y - Sy)\|^2, \ \forall x, y \in C.$$

If $k = 0$, then S is said to be *nonexpansive*.

(ii) *nonspreading* [17] if $2\|Sx - Sy\|^2 \leq \|Sx - y\|^2 + \|Sy - x\|^2$, for all $x, y \in C$.

(iii) (α, β)*-generalized hybrid* [16] if there exist real numbers α, β such that

$$\alpha\|Sx - Sy\|^2 + (1 - \alpha)\|x - Sy\|^2 \leq \beta\|Sx - y\|^2 + (1 - \beta)\|x - y\|^2, \ \forall x, y \in C.$$

(iv) ψ*-demimetric* [33] if $F(S) \neq \emptyset$ and there exists $\psi \in (-\infty, 1)$ such that

$$2\langle x - p, x - Sx\rangle \geq (1 - \psi)\|x - Sx\|^2, \ \forall x \in C, p \in F(S).$$

(v) *averaged* if it can be written as $S = (1 - \alpha)I + \alpha T$, where $\alpha \in (0,1)$, $T : C \to C$ is nonexpansive and I is the identity mapping on C;

(vi) β*-inverse strongly monotone* (β-ism) if there exits $\beta > 0$, such that

$$\langle Sx - Sy, x - y\rangle \geq \beta||Sx - Sy||^2, \ \forall \ x, y \ \in C.$$

From the above definitions, we observe that a k-strict pseudo-contraction S with $F(S) \neq \emptyset$ is k-demimetric, an (α, β)-generalized hybrid mapping S with $F(S) \neq \emptyset$ is 0-demimetric and the metric resolvent J_λ^A with $A^{-1}0 \neq \emptyset$ is (-1)-demimetric. We have the following result on demimetric mappings.

Lemma 2.1. *Let $\psi \in (-\infty, 1)$ and let $T : C \to H$ be a ψ-demimetric mapping such that $F(S) \neq \emptyset$. Let μ be a real number with $0 < \mu \leq 1 - \psi$ and define $U = (1 - \mu)I + \mu T$. Then U is a quasi-nonexpansive mapping of C into H.*

Let X be a nonempty subset of a real Hilbert space H. Recall that a function $f : X \to (-\infty, +\infty]$ is said to be

(i) proper if its effective domain $D(f) = \{x \in X : f(x) < +\infty\}$ is non-empty,
(ii) convex if $f(\lambda x + (1 - \lambda)y) \leq \lambda f(x) + (1 - \lambda) f(y)$, for all $\lambda \in (0,1)$, and $x, y \in X$,
(iii) lower semi-continuous at $x_0 \in D(f)$ if $f(x_0) \leq \lim_{x \to x_0} f(x)$.

Let $f : H \to (-\infty, \infty]$ be proper. A subdifferential ∂f of f is defined by

$$\partial f(x) := \{v \in H : f(x) + \langle v, y - x\rangle \leq f(y), \ x, y \in H\}$$

The set of minimizers of f denoted by Argmin f is $\operatorname{Arg\,min} f := \{x \in H : f(x) \leq f(y), \forall y \in H\}$. It is known that if f is proper, lower semicontinuous and convex, then ∂f is a maximal monotone operator (see [29]). Thus, the resolvent of ∂f denoted by $J_\lambda^{\partial f}$ is called the proximity operator [2] of λf and is denoted by $Prox_{\lambda f}$.

Example 2.1. Let $i_C : H \to (-\infty, +\infty]$ of C be defined by $i_C(x) := \begin{cases} 0, & \text{if } x \in C, \\ \infty, & \text{if } x \notin C. \end{cases}$ Clearly,

$$\partial i_C(x) = N_C(x) := \begin{cases} \{u \in H : \langle u, v-x\rangle \leq 0,\ v \in C\}, & \text{if } x \in C, \\ \emptyset & \text{otherwise,} \end{cases}$$

where $N_C(x)$ is the normal cone to C. Since i_C is a proper, lower semicontinuous, and convex function, then $\partial i_C = N_C$ is maximal monotone, and $J_\mu^{\partial i_C} = J_\mu^{N_C} = P_C$, for all $\mu > 0$.

Lemma 2.2. [19] *Let $\{a_n\}, \{c_n\} \subset \mathbb{R}_+, \{\sigma_n\} \subset (0,1)$ and $\{b_n\} \subset \mathbb{R}$ be sequences such that $a_{n+1} \leq (1-\sigma_n)a_n + b_n + c_n$. Assume $\sum_{n=0}^{\infty} |c_n| < \infty$. (1) If $b_n \leq \beta\sigma_n$ for some $\beta \geq 0$, then $\{a_n\}$ is a bounded sequence; (2) If $\sum_{n=0}^{\infty} \sigma_n = \infty$ and $\limsup_{n\to\infty} \frac{b_n}{\sigma_n} \leq 0$, then $\lim_{n\to\infty} a_n = 0$.*

Lemma 2.3. [30] *Let $\{a_n\}$ be a sequence of non-negative real numbers, $\{\alpha_n\}$ be a sequence in $(0,1)$ with $\sum_{n=1}^{\infty} \alpha_n = \infty$, and $\{b_n\}$ be a sequence of real numbers. Let $a_{n+1} \leq (1-\alpha_n)a_n + \alpha_n b_n$. If $\limsup_{k\to\infty} b_{n_k} \leq 0$ for every subsequence $\{a_{n_k}\}$ of $\{a_n\}$ satisfying $\liminf_{k\to\infty}(a_{n_{k+1}} - a_{n_k}) \geq 0$, then $\lim_{n\to\infty} a_n = 0$.*

Lemma 2.4. [39] *Let $T : H \to H$ be a given operator. (i) T is firmly nonexpansive if and only if its complement $I - T$ is firmly nonexpansive. (ii) T is averaged if and only if its complement $I - T$ is β-ism for some $\beta > \frac{1}{2}$. Indeed, for $\alpha \in (0,1), T$ is α-averaged if and only if $I - T$ is $\frac{1}{2\alpha}$-ism.*

3. The Algorithm

In this section, we present our proposed algorithm for solving split null point problems for maximal monotone operators and fixed point problems for ψ-demimetric mappings in Hilbert spaces, and we highlight some of its important features.

Let H_1 and H_2 be two real Hilbert spaces, and let $A : H_1 \to H_2$ be a linear and bounded mapping. Suppose that $B_1 : H_1 \to 2^{H_1}$ and $B_2 : H_2 \to 2^{H_2}$ are maximal monotone mappings and let $J_\lambda^{B_1}$ and $J_\lambda^{B_2}$ be the resolvents of B_1 and B_2, respectively for $\lambda > 0$. Let $T : H_1 \to H_1$ be a demimetric mapping with constant ψ such that $I - T$ is demiclosed at zero. Let the solution set be denoted by $\Gamma = F(T) \cap \Omega \neq \emptyset$, where $\Omega = \{x \in B_1^{-1}0 : Ax \in B_2^{-1}0\}$ and let $f : H_1 \to H_1$ be a μ-contraction mapping. We establish our results under the following conditions on the control parameters:

(C1) $\lambda_n \in (0, \infty)$ such that $\liminf_{n\to\infty} \lambda_n > 0$;
(C2) $\alpha_n \in (0,1)$ such that $\lim_{n\to\infty} \alpha_n = 0$ and $\sum_{n=1}^{\infty} \alpha_n = \infty$;
(C3) $0 < a \leq \sigma_n \leq 1 - \psi, 0 < b \leq \beta_n, \xi_n \leq 1$, and $0 < c \leq \tau_n \leq d < 2$;
(C4) Let $\theta > 0, \{\delta_n\}$ be a positive sequence such that $\lim_{n\to\infty} \frac{\delta_n}{\alpha_n} = 0$.

Now, our proposed algorithm is presented as follows.

Algorithm 3.1.

Step 0. Let $x_0, x_1 \in H_1$ be two arbitrary initial points and set $n = 1$.
Step 1. Given the $(n-1)th$ and nth iterates, choose θ_n such that $0 \leq \theta_n \leq \hat{\theta}_n$ with $\hat{\theta}_n$ defined by

$$\hat{\theta}_n = \begin{cases} \min\left\{\theta,\ \frac{\delta_n}{\|x_n - x_{n-1}\|}\right\}, & \text{if } x_n \neq x_{n-1}, \\ \theta, & \text{otherwise.} \end{cases} \tag{3.1}$$

Step 2. Compute $w_n = x_n + \theta_n(x_n - x_{n-1})$.
Step 3. Compute $y_n = J_{\lambda_n}^{B_1}(w_n + \gamma_n A^*(J_{\lambda_n}^{B_2} - I)Aw_n)$, where γ_n is defined by

$$\gamma_n = \begin{cases} \tau_n \frac{||(J_{\lambda_n}^{B_2}-I)Aw_n||^2}{||A^*(J_{\lambda_n}^{B_2}-I)Aw_n||^2} & \text{If } Aw_n \neq J_{\lambda_n}^{B_2}Aw_n, \\ \gamma, & \text{otherwise } (\gamma \text{ is any nonegative real number}). \end{cases} \tag{3.2}$$

Step 4. Compute $u_n = (1-\sigma_n)y_n + \sigma_n T y_n$.
Step 5. Compute $x_{n+1} = \alpha_n f(w_n) + \beta_n y_n + \xi_n u_n$.
Set $n := n+1$ and return to **Step 1**.

Remark 3.1. From conditions (C2) and (C4), one can easily verify from (3.1) that

$$\lim_{n\to\infty} \theta_n ||x_n - x_{n-1}|| = 0 \text{ and } \lim_{n\to\infty} \frac{\theta_n}{\alpha_n} ||x_n - x_{n-1}|| = 0.$$

Remark 3.2. We observe that the implementation of our proposed algorithm does not require prior knowledge of the operator norm unlike Algorithm 1.1, 1.2, 1, and 1. Hence, this makes our algorithm easily implementable. Moreover, we employ the inertial technique to accelerate the rate of convergence. Our result improves on Byrne et al. [5], Jailoka and Suantai [12], Jailoka et al. [13], and Takahashi et al. [32].

Lemma 3.1. *The step size $\{\gamma_n\}$ of the Algorithm 3.1 defined by (3.2) is well defined.*

Proof. Let $p \in \Omega$. From Remark 2.1(3), we have that $J_{\lambda_n}^{B_1} p = p, J_{\lambda_n}^{B_2} Ap = Ap$. By applying the fact $J_{\lambda_n}^{B_2}$ is averaged together with Lemma 2.4(ii), we have

$$\begin{aligned} \|A^*(I - J_{\lambda_n}^{B_2})Aw_n\| \|w_n - p\| &\geq \langle A^*(I - J_{\lambda_n}^{B_2})Aw_n, w_n - p\rangle \\ &= \langle (I - J_{\lambda_n}^{B_2})Aw_n - (I - J_{\lambda_n}^{B_2})Ap, Aw_n - Ap\rangle \\ &\geq \beta \|(I - J_{\lambda_n}^{B_2})Aw_n\|^2, \end{aligned}$$

for some $\beta > \frac{1}{2}$. It follows that $\|A^*(I - J_{\lambda_n}^{B_2})Aw_n\| > 0$ when $\|(I - J_{\lambda_n}^{B_2})Aw_n\| \neq 0$. Hence, $\{\gamma_n\}$ is well defined.

□

4. Convergence Analysis

Now, we analyze the convergence of our proposed algorithm. First, we establish the following lemmas.

Lemma 4.1. *Let $\{x_n\}$ be a sequence generated by Algorithm 3.1. Then $\{x_n\}$ is bounded.*

Proof. Let $p \in \Gamma$. Then $J_{\lambda_n}^{B_1} p = p, J_{\lambda_n}^{B_2} Ap = Ap$ and $Tp = p$. Thus

$$\|w_n - p\| = \|x_n + \theta_n(x_n - x_{n-1}) - p\| \leq \|x_n - p\| + \alpha_n \frac{\theta_n}{\alpha_n} \|x_n - x_{n-1}\|.$$

In view of condition (C4), one sees that there exists $M_1 > 0$ such that $\frac{\theta_n}{\alpha_n}\|x_n - x_{n-1}\| \leq M_1$. Hence, $\|w_n - p\| \leq \|x_n - p\| + \alpha_n M_1$. Since T is ψ-demimetric and $0 < a \leq \sigma_n \leq 1 - \psi$, Lemma

2.1 yields $\|u_n - p\| \leq \|[(1-\sigma_n)I + \sigma_n T]y_n - p\| \leq \|y_n - p\|$. From the firm nonexpansiveness of $J_{\lambda_n}^{B_1}$ and $I - J_{\lambda_n}^{B_1}$, the definition of γ_n together with the condition on τ_n, we obtain

$$\begin{aligned}
\|y_n - p\|^2 &\leq \|w_n + \gamma_n A^*(J_{\lambda_n}^{B_2} - I)Aw_n - p\|^2 \\
&= \|w_n - p\|^2 + \gamma_n^2 \|A^*(J_{\lambda_n}^{B_2} - I)Aw_n\|^2 + 2\gamma_n \langle w_n - p, A^*(J_{\lambda_n}^{B_2} - I)Aw_n\rangle \\
&= \|w_n - p\|^2 + \gamma_n^2 \|A^*(J_{\lambda_n}^{B_2} - I)Aw_n\|^2 + 2\gamma_n \langle Aw_n - Ap, (J_{\lambda_n}^{B_2} - I)Aw_n - (J_{\lambda_n}^{B_2} - I)Ap\rangle \\
&\leq \|w_n - p\|^2 - \gamma_n(2-\gamma_n)\|(J_{\lambda_n}^{B_2} - I)Aw_n\|^2 \\
&\leq \|w_n - p\|^2.
\end{aligned} \tag{4.1}$$

Thus

$$\begin{aligned}
\|x_{n+1} - p\| &\leq \alpha_n \mu \|w_n - p\| + \alpha_n \|f(p) - p\| + \beta_n \|y_n - p\| + \xi_n \|u_n - p\| \\
&\leq \big(\alpha_n \mu + (1-\alpha_n)\big)\|w_n - p\| + \alpha_n \|f(p) - p\| \\
&= \big(\alpha_n \mu + (1-\alpha_n)\big)(\|x_n - p\| + \alpha_n M_1) + \alpha_n \|f(p) - p\| \\
&= \big(1 - \alpha_n(1-\mu)\big)\|x_n - p\| + \alpha_n(1-\mu)\left\{\frac{\|f(p) - p\|}{1-\mu} + \frac{\big(1-\alpha_n(1-\mu)\big)}{1-\mu} M_1\right\} \\
&\leq \big(1 - \alpha_n(1-\mu)\big)\|x_n - p\| + \alpha_n(1-\mu)M^*,
\end{aligned}$$

where

$$M^* = \sup_{n \in \mathbb{N}} \left\{\frac{\|f(p) - p\|}{1-\mu} + \frac{\big(1-\alpha_n(1-\mu)\big)}{1-\mu} M_1\right\}.$$

Set $a_n := ||x_n - p||$; $b_n := \alpha_n(1-\mu)M^*$; $c_n := 0$, and $\sigma_n := \alpha_n(1-\mu)$. Using Lemma 2.2 and the assumptions on the control parameters, we have that $\{||x_n - p||\}$ is bounded and this implies that $\{x_n\}$ is bounded. It follows from the construction of the algorithm that $\{w_n\}, \{y_n\}$, and $\{u_n\}$ are also bounded. This completes the proof. □

Lemma 4.2. *Let $\{x_n\}$ be a sequence generated by Algorithm 3.1. Then the following inequality holds for all $p \in \Gamma$*

$$\begin{aligned}
\|x_{n+1} - p\|^2 \leq \left(1 - \frac{2\alpha_n(1-\mu)}{(1-\alpha_n\mu)}\right)\|x_n - p\|^2 + \frac{2\alpha_n(1-\mu)}{(1-\alpha_n\mu)}\Bigg\{&\frac{\alpha_n}{2(1-\mu)}M_2 \\
&+ \frac{3M\big((1-\alpha_n)^2 + \alpha_n\mu\big)}{2(1-\mu)}\frac{\theta_n}{\alpha_n}\|x_n - x_{n-1}\| \\
&+ \frac{1}{(1-\mu)}\langle f(p) - p, x_{n+1} - p\rangle\Bigg\} - \frac{(1-\alpha_n)^2\gamma_n(2-\tau_n)}{(1-\alpha_n\mu)}\|(J_{\lambda_n}^{B_2} - I)Aw_n\|^2.
\end{aligned}$$

Proof. Note that

$$\begin{aligned}
\|w_n - p\|^2 &= \|x_n - p\|^2 + \theta_n^2\|x_n - x_{n-1}\|^2 + 2\theta_n\langle x_n - p, x_n - x_{n-1}\rangle \\
&\leq \|x_n - p\|^2 + \theta_n\|x_n - x_{n-1}\|\big(2\|x_n - p\| + \theta_n\|x_n - x_{n-1}\|\big) \\
&\leq \|x_n - p\|^2 + 3M\alpha_n\frac{\theta_n}{\alpha_n}\|x_n - x_{n-1}\|.
\end{aligned} \tag{4.2}$$

where $M := \sup_{n\in\mathbb{N}}\{\|x_n - p\|, \theta\|x_n - x_{n-1}\|\}$. It follows that

$$\begin{aligned}
&\|x_{n+1}-p\|^2\\
&\le \beta_n^2\|y_n-p\|^2+\xi_n^2\|u_n-p\|^2+2\beta_n\xi_n\|y_n-p\|\|u_n-p\|\\
&\quad +2\alpha_n\langle f(w_n)-f(p),x_{n+1}-p\rangle+2\alpha_n\langle f(p)-p,x_{n+1}-p\rangle\\
&\le \beta_n(\beta_n+\xi_n)\|y_n-p\|^2+\xi_n(\xi_n+\beta_n)\|u_n-p\|^2+\alpha_n\mu\left[\|w_n-p\|^2+\|x_{n+1}-p\|^2\right]\\
&\quad +2\alpha_n\langle f(p)-p,x_{n+1}-p\rangle\\
&\le \beta_n(1-\alpha_n)\|y_n-p\|^2+\xi_n(1-\alpha_n)\|y_n-p\|^2+\alpha_n\mu\left[\|w_n-p\|^2+\|x_{n+1}-p\|^2\right]\\
&\quad +2\alpha_n\langle f(p)-p,x_{n+1}-p\rangle\\
&\le \left((1-\alpha_n)^2+\alpha_n\mu\right)\|w_n-p\|^2+\alpha_n\mu\|x_{n+1}-p\|^2+2\alpha_n\langle f(p)-p,x_{n+1}-p\rangle\\
&-(1-\alpha_n)^2\gamma_n(2-\tau_n)\|(J_{\lambda_n}^{B_2}-I)Aw_n\|^2\\
&\le \left((1-\alpha_n)^2+\alpha_n\mu\right)\left[\|x_n-p\|^2+3M\alpha_n\frac{\theta_n}{\alpha_n}\|x_n-x_{n-1}\|\right]+\alpha_n\mu\|x_{n+1}-p\|^2\\
&\quad +2\alpha_n\langle f(p)-p,x_{n+1}-p\rangle-(1-\alpha_n)^2\gamma_n(2-\tau_n)\|(J_{\lambda_n}^{B_2}-I)Aw_n\|^2,
\end{aligned}$$

which implies that

$$\begin{aligned}
&\|x_{n+1}-p\|^2\\
&\le \frac{(1-2\alpha_n+\alpha_n\mu)}{1-\alpha_n\mu}\|x_n-p\|^2+\frac{\alpha_n^2}{(1-\alpha_n\mu)}\|x_n-p\|^2+3M\frac{\left((1-\alpha_n)^2+\alpha_n\mu\right)}{(1-\alpha_n\mu)}\alpha_n\frac{\theta_n}{\alpha_n}\|x_n-x_{n-1}\|\\
&+\frac{2\alpha_n}{(1-\alpha_n\mu)}\langle f(p)-p,x_{n+1}-p\rangle-\frac{(1-\alpha_n)^2\gamma_n(2-\tau_n)\|(J_{\lambda_n}^{B_2}-I)Aw_n\|^2}{(1-\alpha_n\mu)}\\
&\le \left(1-\frac{2\alpha_n(1-\mu)}{(1-\alpha_n\mu)}\right)\|x_n-p\|^2+\frac{2\alpha_n(1-\mu)}{(1-\alpha_n\mu)}\left\{\frac{3M\left((1-\alpha_n)^2+\alpha_n\mu\right)}{2(1-\mu)}\frac{\theta_n}{\alpha_n}\|x_n-x_{n-1}\|\right.\\
&\left.+\frac{\alpha_n}{2(1-\mu)}M_2+\frac{1}{(1-\mu)}\langle f(p)-p,x_{n+1}-p\rangle\right\}-\frac{(1-\alpha_n)^2\gamma_n(2-\tau_n)}{(1-\alpha_n\mu)}\|(J_{\lambda_n}^{B_2}-I)Aw_n\|^2,
\end{aligned}$$

where $M_2 = \sup\{\|x_n - p\|^2 : n \in \mathbb{N}\}$. This complete the proof. □

Lemma 4.3. *The following inequality holds for all $p \in \Gamma$ and $n \in \mathbb{N}$*

$$\begin{aligned}
||x_{n+1}-p||^2 &\le (1-\alpha_n)\|x_n-p\|^2+\alpha_n\left[\|f(w_n)-p\|^2+3M(1-\alpha_n)\frac{\theta_n}{\alpha_n}\|x_n-x_{n-1}\|\right]\\
&\quad +2M_3(1-\alpha_n)\|A^*(J_{\lambda_n}^{B_2}-I)Aw_n\|-(1-\alpha_n)\|y_n-w_n\|^2-\beta_n\xi_n\|y_n-u_n\|^2.
\end{aligned}$$

Proof. Let $p \in \Gamma$. From (4.1), we see that $||w_n + \gamma_n A^*(J_{\lambda_n}^{B_2} - I)Aw_n - p||^2 \leq ||w_n - p||^2$. Applying the firm nonexpansivity of $J_{\lambda_n}^{B_1}$, we arrive at

$$\begin{aligned}
||y_n - p||^2 &\leq \langle y_n - p, w_n + \gamma_n A^*(J_{\lambda_n}^{B_2} - I)Aw_n - p\rangle \\
&\leq \frac{1}{2}\big\{||y_n - p||^2 + ||w_n - p||^2 - \big(||y_n - w_n||^2 + \gamma_n^2||A^*(J_{\lambda_n}^{B_2} - I)Aw_n||^2 \\
&\quad - 2\gamma_n\langle y_n - w_n, A^*(J_{\lambda_n}^{B_2} - I)Aw_n\rangle\big)\big\} \\
&\leq \frac{1}{2}\big\{||y_n - p||^2 + ||w_n - p||^2 - ||y_n - w_n||^2 - \gamma_n^2||A^*(J_{\lambda_n}^{B_2} - I)Aw_n||^2 \\
&\quad + 2\gamma_n||y_n - w_n||||A^*(J_{\lambda_n}^{B_2} - I)Aw_n||\big\} \\
&\leq \frac{1}{2}\big\{||y_n - p||^2 + ||w_n - p||^2 - ||y_n - w_n||^2 + 2\gamma_n||y_n - w_n||||A^*(J_{\lambda_n}^{B_2} - I)Aw_n||\big\},
\end{aligned}$$

which implies that $\|y_n - p||^2 \leq ||w_n - p||^2 - ||y_n - w_n||^2 + 2M_3||A^*(J_{\lambda_n}^{B_2} - I)Aw_n||$, where $M_3 := \sup\{\gamma_n||y_n - w_n|| : n \in \mathbb{N}\}$. Using (4.2), we obtain

$$\begin{aligned}
\|x_{n+1} - p\|^2 &\leq \alpha_n\|f(w_n) - p\|^2 + \beta_n\|y_n - p\|^2 + \xi_n\|u_n - p\|^2 - \beta_n\xi_n\|y_n - u_n\|^2 \\
&\leq \alpha_n\|f(w_n) - p\|^2 + \beta_n\|y_n - p\|^2 + \xi\|y_n - p\|^2 - \beta_n\xi_n\|y_n - u_n\|^2 \\
&\leq \alpha_n\|f(w_n) - p\|^2 + (1 - \alpha_n)\big[\|w_n - p\|^2 - \|y_n - w_n\|^2 + 2M_3\|A^*(J_{\lambda_n}^{B_2} - I)Aw_n\|\big] \\
&\quad - \beta_n\xi_n\|y_n - u_n\|^2 \\
&\leq (1 - \alpha_n)\|x_n - p\|^2 + \alpha_n\big[\|f(w_n) - p\|^2 + 3M(1 - \alpha_n)\frac{\theta_n}{\alpha_n}\|x_n - x_{n-1}\|\big] \\
&\quad + 2M_3(1 - \alpha_n)\|A^*(J_{\lambda_n}^{B_2} - I)Aw_n\| - (1 - \alpha_n)\|y_n - w_n\|^2 - \beta_n\xi_n\|y_n - u_n\|^2,
\end{aligned}$$

which is the required inequality. □

Theorem 4.1. *Let H_1 and H_2 be two real Hilbert spaces, and let $A : H_1 \to H_2$ be a linear and bounded mapping. Let $B_1 : H_1 \to 2^{H_1}$ and $B_2 : H_2 \to 2^{H_2}$ be maximal monotone mappings, and let $J_\lambda^{B_1}$ and $J_\lambda^{B_2}$ be resolvents of B_1 and B_2, respectively for $\lambda > 0$. Let $T : H_1 \to H_1$ be a demimetric mapping with constant ψ such that $I - T$ is demiclosed at zero. Let $f : H_1 \to H_1$ be a μ-contraction mapping and suppose that the solution set denoted by $\Gamma = F(T) \cap \Omega \neq \emptyset$, where $\Omega = \{x \in B_1^{-1}0 : Ax \in B_2^{-1}0\}$. Let $\{x_n\}$ be a sequence generated by Algorithm 3.1 such that conditions (C1) - (C4) are satisfied. Then, $\{x_n\}$ converges strongly to a point $\hat{x} \in \Gamma$, where $\hat{x} = P_\Gamma \circ f(\hat{x})$.*

Proof. Let $\hat{x} = P_\Gamma \circ f(\hat{x})$. From Lemma 4.2, we obtain

$$\begin{aligned}
\|x_{n+1} - \hat{x}\|^2 \leq \frac{2\alpha_n(1-\mu)}{(1-\alpha_n\mu)}\bigg\{&\frac{\alpha_n}{2(1-\mu)}M_2 + \frac{3M\big((1-\alpha_n)^2 + \alpha_n\mu\big)}{2(1-\mu)}\frac{\theta_n}{\alpha_n}\|x_n - x_{n-1}\| \\
&+ \frac{1}{(1-\mu)}\langle f(\hat{x}) - \hat{x}, x_{n+1} - \hat{x}\rangle\bigg\} + \left(1 - \frac{2\alpha_n(1-\mu)}{(1-\alpha_n\mu)}\right)\|x_n - \hat{x}\|^2. \qquad (4.3)
\end{aligned}$$

Next, we claim that the sequence $\{\|x_n - \hat{x}\|\}$ converges to zero. To establish this, by Lemma 2.3, it suffices to show that $\limsup_{k\to\infty}\langle f(\hat{x}) - \hat{x}, x_{n_k+1} - \hat{x}\rangle \leq 0$ for every subsequence $\{\|x_{n_k} -$

$\hat{x}\|\}$ of $\{\|x_n-\hat{x}\|\}$ satisfying

$$\liminf_{k\to\infty}(\|x_{n_k+1}-\hat{x}\|-\|x_{n_k}-\hat{x}\|)\geq 0. \tag{4.4}$$

Suppose that $\{\|x_{n_k}-\hat{x}\|\}$ is a subsequence of $\{\|x_n-\hat{x}\|\}$ such that (4.4) holds. In view of Lemma 4.2, we have

$$\begin{aligned}&\frac{(1-\alpha_{n_k})^2\gamma_{n_k}(1-\tau_{n_k})}{(1-\alpha_{n_k}\mu)}\|(J^{B_2}_{\lambda_{n_k}}-I)Aw_{n_k}\|^2\\&\leq\left(1-\frac{2\alpha_{n_k}(1-\mu)}{(1-\alpha_{n_k}\mu)}\right)\|x_{n_k}-\hat{x}\|^2-\|x_{n_k+1}-\hat{x}\|^2+\frac{2\alpha_{n_k}(1-\mu)}{(1-\alpha_{n_k}\mu)}\left\{\frac{\alpha_{n_k}}{2(1-\mu)}M_2\right.\\&\quad\left.+\frac{3M\big((1-\alpha_{n_k})^2+\alpha_{n_k}\mu\big)}{2(1-\mu)}\frac{\theta_{n_k}}{\alpha_{n_k}}\|x_{n_k}-x_{n_k-1}\|+\frac{1}{(1-\mu)}\langle f(\hat{x})-\hat{x},x_{n_k+1}-\hat{x}\rangle\right\}.\end{aligned}$$

By (4.4) and the condition on α_{n_k}, we obtain

$$\frac{(1-\alpha_{n_k})^2\gamma_{n_k}(2-\tau_{n_k})}{(1-\alpha_{n_k}\mu)}\|(J^{B_2}_{\lambda_{n_k}}-I)Aw_{n_k}\|^2\to 0,\ \text{as}\ k\to\infty.$$

In view of the definition of γ_n, we have

$$\frac{(1-\alpha_{n_k})^2\tau_{n_k}(2-\tau_{n_k})\|(J^{B_2}_{\lambda_{n_k}}-I)Aw_{n_k}\|^4}{\|A^*(J^{B_2}_{\lambda_{n_k}}-I)Aw_{n_k}\|^2}\to 0,\ \text{as}\ k\to\infty,$$

which implies that

$$\frac{\|(J^{B_2}_{\lambda_{n_k}}-I)Aw_{n_k}\|^2}{\|A^*(J^{B_2}_{\lambda_{n_k}}-I)Aw_{n_k}\|}\to 0,\ \text{as}\ k\to\infty.$$

Note that $\|A^*(J^{B_2}_{\lambda_{n_k}}-I)Aw_{n_k}\|\leq\|A^*\|\|(J^{B_2}_{\lambda_{n_k}}-I)Aw_{n_k}\|=\|A\|\|(J^{B_2}_{\lambda_{n_k}}-I)Aw_{n_k}\|$. It follows that

$$\|(J^{B_2}_{\lambda_{n_k}}-I)Aw_{n_k}\|\leq\|A\|\frac{\|(J^{B_2}_{\lambda_{n_k}}-I)Aw_{n_k}\|^2}{\|A^*(J^{B_2}_{\lambda_{n_k}}-I)Aw_{n_k}\|}.$$

Consequently, we obtain

$$\|(J^{B_2}_{\lambda_{n_k}}-I)Aw_{n_k}\|\to 0,\ \text{as}\ k\to\infty, \tag{4.5}$$

and

$$\|A^*(J^{B_2}_{\lambda_{n_k}}-I)Aw_{n_k}\|\to 0,\ \text{as}\ k\to\infty. \tag{4.6}$$

Lemma 4.3 yields

$$\begin{aligned}&(1-\alpha_n)\|y_{n_k}-w_{n_k}\|^2+\beta_{n_k}\xi_{n_k}\|y_{n_k}-u_{n_k}\|^2\\&\leq(1-\alpha_{n_k})\|x_{n_k}-p\|^2-\|x_{n_k+1}-p\|^2+\alpha_{n_k}\Big[\|f(w_{n_k})-p\|^2\\&\quad+3M(1-\alpha_{n_k})\frac{\theta_{n_k}}{\alpha_{n_k}}\|x_{n_k}-x_{n_k-1}\|\Big]+2M_3(1-\alpha_{n_k})\|A^*(J^{B_2}_{\lambda_{n_k}}-I)Aw_{n_k}\|.\end{aligned}$$

By (4.4), (4.6) and the conditions on the control parameters, we see that

$$\lim_{k\to\infty}\|y_{n_k}-w_{n_k}\|=\lim_{k\to\infty}\|y_{n_k}-u_{n_k}\|=0, \tag{4.7}$$

which implies $\lim_{k\to\infty}\|w_{n_k}-u_{n_k}\|=0$. Note that $\|w_{n_k}-x_{n_k}\|=\theta_{n_k}\|x_{n_k}-x_{n_k-1}\|$. By Remark 3.1, we see that

$$\lim_{k\to\infty}\|w_{n_k}-x_{n_k}\|=0. \tag{4.8}$$

From (4.7) and (4.8), we obtain

$$\lim_{k\to\infty}\|x_{n_k}-y_{n_k}\|=\lim_{k\to\infty}\|x_{n_k}-u_{n_k}\|=0. \tag{4.9}$$

Since T is ψ-demimetric, for $\hat{x}\in\Gamma$, we have

$$\langle y_{n_k}-\hat{x},y_{n_k}-u_{n_k}\rangle=\sigma_{n_k}\langle y_{n_k}-\hat{x},y_{n_k}-Ty_{n_k}\rangle\geq\frac{\sigma_{n_k}(1-\psi)}{2}\|y_{n_k}-Ty_{n_k}\|^2.$$

It follows from $\lim_{k\to\infty}\|y_{n_k}-u_{n_k}\|=0$ that

$$\lim_{k\to\infty}\|y_{n_k}-Ty_{n_k}\|=0. \tag{4.10}$$

Next, applying (4.9) together with the fact that $\lim_{k\to\infty}\alpha_{n_k}=0$, we have

$$\|x_{n_k+1}-x_{n_k}\|\leq\alpha_n\|f(w_{n_k})-x_{n_k}\|+\beta_{n_k}\|y_{n_k}-x_{n_k}\|+\xi_{n_k}\|u_{n_k}-x_{n_k}\|\to 0,\quad k\to\infty. \tag{4.11}$$

Since $\{x_n\}$ is bounded, then $w_\omega(x_n)$ is nonempty. Let $\bar{x}\in w_\omega(x_n)$ be an arbitrary element. Then, there exists a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ such that $x_{n_k}\rightharpoonup\bar{x}$ as $k\to\infty$. From (4.9), it follows that $y_{n_k}\rightharpoonup\bar{x}$ as $k\to\infty$. Since $I-T$ is demiclosed at zero, we obtain from (4.10) that $\bar{x}\in F(T)$. Note that A is linear and bounded and $w_{n_k}\rightharpoonup\bar{x}$ by (4.8). Then it follows that $Aw_{n_k}\rightharpoonup A\bar{x}$. By the demiclosedness of $I-J_{\lambda_n}^{B_2}$ and (4.5), we obtain $A\bar{x}\in B_2^{-1}0$. Let $(s,t)\in G(B_1)$. From $y_{n_k}=J_{\lambda_{n_k}}^{B_1}(w_{n_k}+\gamma_{n_k}A^*(J_{\lambda_{n_k}}^{B_2}-I)Aw_{n_k})$ and the fact that B_1 is a maximal monotone operator, we have

$$\left\langle s-y_{n_k},t-\frac{(w_{n_k}-y_{n_k})+\gamma_{n_k}A^*(J_{\lambda_{n_k}}^{B_2}-I)Aw_{n_k}}{\lambda_{n_k}}\right\rangle\geq 0,$$

which yields

$$\langle s-y_{n_k},t\rangle\geq\left\langle s-y_{n_k},\frac{w_{n_k}-y_{n_k}}{\lambda_{n_k}}\right\rangle+\left\langle s-y_{n_k},\frac{\gamma_{n_k}A^*(J_{\lambda_{n_k}}^{B_2}-I)Aw_{n_k}}{\lambda_{n_k}}\right\rangle. \tag{4.12}$$

Since $y_{n_k}\rightharpoonup\bar{x}$, we obtain $\lim_{k\to\infty}\langle s-y_{n_k},t\rangle=\langle s-\bar{x},t\rangle$. By applying (4.6) and (4.7) to (4.12) together with condition (C1), we obtain $\langle s-\bar{x},t\rangle\geq 0$. By the maximal monotonicity of B_1, we have $\bar{x}\in B_1^{-1}0$. Hence, $w_\omega(x_n)\subset\Gamma$.

Next, from (4.9), we see that $w_\omega\{y_{n_k}\}=w_\omega\{x_{n_k}\}$. By the boundedness of $\{x_{n_k}\}$, there exists a subsequence $\{x_{n_{k_j}}\}$ of $\{x_{n_k}\}$ such that $x_{n_{k_j}}\rightharpoonup x^\dagger$ and

$$\lim_{j\to\infty}\langle f(\hat{x})-\hat{x},x_{n_{k_j}}-\hat{x}\rangle=\limsup_{k\to\infty}\langle f(\hat{x})-\hat{x},x_{n_k}-\hat{x}\rangle=\limsup_{k\to\infty}\langle f(\hat{x})-\hat{x},y_{n_k}-\hat{x}\rangle.$$

Since $\hat{x}=P_\Gamma\circ f(\hat{x})$, it follows that

$$\limsup_{k\to\infty}\langle f(\hat{x})-\hat{x},x_{n_k}-\hat{x}\rangle=\lim_{j\to\infty}\langle f(\hat{x})-\hat{x},x_{n_{k_j}}-\hat{x}\rangle=\langle f(\hat{x})-\hat{x},x^\dagger-\hat{x}\rangle\leq 0.$$

In view of (4.11), we obtain $\limsup_{k\to\infty}\langle f(\hat{x})-\hat{x},x_{n_k+1}-\hat{x}\rangle\leq 0$. Applying Lemma 2.3 to (4.3), and using Remark 3.1 and the fact that $\lim_{n\to\infty}\alpha_n=0$, it follows that $\lim_{n\to\infty}\|x_n-\hat{x}\|=0$ as required. □

5. Application

5.1. **Split minimization problem.** Let $f_1 : H_1 \to (-\infty, \infty]$ and $f_2 : H_2 \to (-\infty, \infty]$ be proper, convex, and lower semicontinuous functions. Let $A : H_1 \to H_2$ be a linear and bounded mapping. The Split Minimization Problem (SMP) is defined to find $\bar{x} \in H_1$ such that

$$\bar{x} \in \text{Argmin } f_1 \text{ and } A\bar{x} \in \text{Argmin } f_2.$$

Now, we have the following result for solving a common solution of the split minimization problem and fixed points of demimetric mappings.

Theorem 5.1. [12] *Let $A : H_1 \to H_2$ be a linear and bounded mapping, and let $T : H_1 \to H_1$ be a $\psi-$demimetric mapping. Let $f_1 : H_1 \to (-\infty, \infty]$ and $f_2 : H_2 \to (-\infty, \infty]$ be proper, convex, and lower semicontinuous functions, and let $\Gamma = F(T) \cap \Omega \neq \emptyset$, where $\Omega = \{x \in \text{Argmin } f_1 : Ax \in \text{Argmin } f_2\}$. Let $f : H_1 \to H_1$ be a $\mu-$contraction mapping, and let $\{x_n\}$ be a sequence generated as*

Algorithm 5.2.

Step 0. Let $x_0, x_1 \in H_1$ be two arbitrary initial points and set $n = 1$.
Step 1. Given the $(n-1)th$ and nth iterates, choose θ_n such that $0 \leq \theta_n \leq \hat{\theta}_n$ with $\hat{\theta}_n$ defined by

$$\hat{\theta}_n = \begin{cases} \min\left\{\theta, \ \frac{\delta_n}{||x_n - x_{n-1}||}\right\}, & \text{if } x_n \neq x_{n-1}, \\ \theta, & \text{otherwise.} \end{cases}$$

Step 2. Compute $w_n = x_n + \theta_n(x_n - x_{n-1})$.
Step 3. Compute $y_n = J_{\lambda_n}^{\partial f_1}(w_n + \gamma_n A^*(J_{\lambda_n}^{\partial f_2} - I)Aw_n)$, where γ_n is defined by

$$\gamma_n = \begin{cases} \tau_n \frac{||(J_{\lambda_n}^{\partial f_2} - I)Aw_n||^2}{||A^*(J_{\lambda_n}^{\partial f_2} - I)Aw_n||^2} & \text{If } Aw_n \neq J_{\lambda_n}^{\partial f_2} Aw_n, \\ \gamma, & \text{otherwise } (\gamma \text{ is any nonegative real number}). \end{cases}$$

Step 4. Compute $u_n = (1 - \sigma_n)y_n + \sigma_n T y_n$.
Step 5. Compute $x_{n+1} = \alpha_n f(w_n) + \beta_n y_n + \xi_n u_n$.
Set $n := n + 1$ and return to **Step 1**.

Suppose that all other conditions of Theorem 4.1 are satisfied. Then, $\{x_n\}$ converges strongly to $\bar{x} \in \Gamma$, where $\hat{x} = P_\Gamma \circ f(\hat{x})$.

Proof. Set $B_1 := \partial f_1$ and $B_2 = \partial f_2$, where B_1 and B_2 are the maximal monotone operators in Theorem 4.1. It is easy to show that Argmin $f_1 = (\partial f_1)^{-1}0 = B_1^{-1}0$ and Argmin $f_2 = (\partial f_2)^{-1}0 = B_2^{-1}0$. Hence, the result follows from Theorem 4.1 immediately. $\square$

5.2. **Split equilibrium problem.** We recall that the equilibrium problem is to find a point $x \in C$ such that $g(x, y) \geq 0$ for all y in C, where C is a nonempty, convex, and closed subset of Hilbert space H and $g : C \times C \to \mathbb{R}$ is a bifunction. We denote its solution set o by $EP(g)$. Suppose that $g : C \times C \to \mathbb{R}$ satisfies the following assumptions:

(A1) for each $x, y, z \in C$, $\limsup_{t \downarrow 0} g(tz + (1-t)x, y) \leq g(x, y)$;
(A2) for all $x \in C$, $g(x, y) + g(y, x) \leq 0$;
(A3) for all $x \in C$, $g(x, x) = 0$:
(A4) for each $x \in C$, $y \mapsto g(x, y)$ is convex and lower semicontinuous.

Let H_1 and H_2 be real Hilbert spaces, and let C and Q be nonempty closed convex subsets of H_1 and H_2, respectively. Let $g_1 : C \times C \to \mathbb{R}$ and $g_2 : Q \times Q \to \mathbb{R}$ be two bifunctions, and let $A : H_1 \to H_2$ be a linear and bounded mapping. The Split Equilibrium Problem (SEP) is to find $\tilde{x} \in C$ such that $g_1(\tilde{x}, x) \geq 0$ for all x in C and $\tilde{y} = A\tilde{x} \in Q$ solves $g_2(\tilde{y}, y) \geq 0$ for all y in$Q$.

Lemma 5.1. [8] *Let $g : C \times C \to \mathbb{R}$ be a bifunction satisfying assumptions (A1)-(A4). For any $r > 0$, define a mapping $T_r^g : H \to C$ by $T_r^g(x) = \{y \in C : g(y,z) + \frac{1}{r}\langle z-y, y-x\rangle \geq 0,\ \forall z \in C\}$. Then, T_r^g is nonempty, single-valued and firmly nonexpansive; and $F(T_r^g) = EP(g)$ is convex and closed.*

Theorem 5.3. [34] *Let $g : C \times C \to \mathbb{R}$ be a bifunction satisfying assumptions (A1)-(A4). Let $A_g : H \to 2^H$ be a multivalued mapping defined by*

$$A_g(x) = \begin{cases} \{y \in H : g(x,z) \geq \langle z-x, y\rangle,\ \forall z \in C\}, & \textit{if } x \in C, \\ \emptyset & \textit{if } x \neq C. \end{cases}$$

Then, A_g is maximal monotone, $EP(g) = A_g^{-1}0$. and $T_r^g = (I + rA_g)^{-1}$ for $r > 0$ i.e. T_r^g is the resolvent of A_g.

Next, we obtain a strong convergence result for solving split equilibrium problems and fixed point problems of demimetric mappings.

Theorem 5.4. [12] *Let $g_1 : C \times C \to \mathbb{R}$ and $g_2 : Q \times Q \to \mathbb{R}$ be bifunctions satisfying (A1)-(A4). Let $A : H_1 \to H_2$ be a linear and bounded mapping. Let $T_r^{g_1}$ and $T_r^{g_2}$ be resolvents of A_{g_1} and A_{g_2} in Theorem 5.3, respectively for $r > 0$. Let $T : H_1 \to H_1$ be a demimetric mapping with constant ψ such that $I - T$ is demiclosed at zero. Suppose $\Gamma = F(T) \cap \Omega \neq \emptyset$, where $\Omega = \{x \in EP(g_1) : Ax \in EP(g_2)\}$ and let $\{x_n\}$ be a sequence generated as*

Algorithm 5.5.

Step 0. Let $x_0, x_1 \in H_1$ be two arbitrary initial points and set $n = 1$.
Step 1. Given the $(n-1)th$ and nth iterates, choose θ_n such that $0 \leq \theta_n \leq \hat{\theta}_n$ with $\hat{\theta}_n$ defined by

$$\hat{\theta}_n = \begin{cases} \min\left\{\theta,\ \frac{\delta_n}{||x_n - x_{n-1}||}\right\}, & \text{if } x_n \neq x_{n-1}, \\ \theta, & \text{otherwise.} \end{cases}$$

Step 2. Compute $w_n = x_n + \theta_n(x_n - x_{n-1})$.
Step 3. Compute $y_n = T_{r_n}^{g_1}(w_n + \gamma_n A^*(T_{r_n}^{g_2} - I)Aw_n)$, where γ_n is defined by

$$\gamma_n = \begin{cases} \tau_n \frac{||(T_{r_n}^{g_2} - I)Aw_n||^2}{||A^*(T_{r_n}^{g_2} - I)Aw_n||^2} & \text{If } Aw_n \neq T_{r_n}^{g_2} Aw_n, \\ \gamma, & \text{otherwise } (\gamma \text{ being any nonegative real number}). \end{cases}$$

Step 4. Compute $u_n = (1 - \sigma_n)y_n + \sigma_n T y_n$.
Step 5. Compute $x_{n+1} = \alpha_n f(w_n) + \beta_n y_n + \xi_n u_n$.
Set $n := n + 1$ and return to **Step 1**.

Suppose that all other conditions of Theorem 4.1 are satisfied. Then, the sequence $\{x_n\}$ converges strongly to $\bar{x} \in \Gamma$, where $\hat{x} = P_\Gamma \circ f(\hat{x})$.

Proof. Let $B_1 := A_{g_1}$ and $B_2 := A_{g_2}$. By Theorem 5.3, set $EP(g_1) = B_1^{-1}0$, $EP(g_2) = B_2^{-1}0$, $T_{r_n}^{g_1} = J_{\lambda_n}^{B_1}$ and $T_{r_n}^{g_2} = J_{\lambda_n}^{B_2}$, where B_1 and B_2 are maximal monotone operators. Then, the result follows from Theorem 4.1 immediately. $\square$

6. Numerical Examples

In this section, we present some numerical experiments to illustrate the performance of our Algorithm 3.1 (Proposed Algo.) in comparison with Algorithm 1.1 (Byrne *et al.* Algo. (1)), Algorithm 1.2 (Byrne *et al.* Algo. (2)), Algorithm 1 (Jailoka *et al.* Algo. (1)) and Algorithm 1 (Jailoka *et al.* Algo. (2)) in the literature. All numerical computations were carried-out using Matlab version R2024a. The numerical results are reported in Figures 1-8 and Tables 1-2. Here, we choose $\alpha_n = \frac{1}{2n+5}$, $\beta_n = \xi_n = \frac{1}{2}(1-\frac{1}{2n+5})$, $\sigma_n = \sigma = \frac{1}{5}, \delta_n = \frac{1}{(2n+5)^3}, \tau_n = \frac{n}{2n+1}, \lambda_n = \lambda = 1, \theta = 0.8$ for each $n \in \mathbb{N}$. We choose $\gamma = 0.05$ in Algorithm 1.1, Algorithm 1.2, Algorithm 1 and Algorithm 1, and we take $r_i = \frac{1}{5}$ in Algorithm 1 for each $i = 1,2$. Moreover, we define $Tx = -\frac{3}{2}x$ in Algorithm 3.1 and we define $S_ix = Sx = -\frac{5}{3}x$ in Algorithm 1 and Algorithm 1.

Example 6.1. Let $H_1 = \mathbb{R}$ and $H_2 = \mathbb{R}^3$ with the usual norms. We define $f(x) = \frac{x}{2}$. Let $g : [-9,3]\times[-9,3] \to \mathbb{R}$ be a function defined by $g(x,y) = y^2 + xy - 2x^2$ and let $h : \mathbb{R}^3 \to (-\infty,\infty]$ be defined by $h(z) = \frac{1}{2}\|Pz\|^2$, where $P = \begin{pmatrix} -3 & 1 & 5 \\ 2 & -4 & 6 \end{pmatrix}$. Let two maximal monotone operators $B_1 : \mathbb{R} \to 2^{\mathbb{R}}$ and $B_2 : \mathbb{R}^3 \to 2^{\mathbb{R}^3}$ be defined by $B_1 := A_g$(see Theorem 5.3) and $B_2 := \partial h$. The explicit resolvents of B_1 and B_2 can be written in the following forms (see [9, 31]):

$$J_1^{B_1}x = \frac{x}{4} \text{ and } J_1^{B_2}z = (P^TP + I)^{-1}z, \ \forall z \in \mathbb{R}^3 \text{ and } x \in \mathbb{R}.$$

Let $A : \mathbb{R} \to \mathbb{R}^3$ be a linear and bounded operator with adjoint A^* defined by

$$Ax = \begin{pmatrix} 2 \\ -5 \\ 3 \end{pmatrix}(x) \quad \text{and} \quad A^*y = \begin{pmatrix} 2 & -5 & 3 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}.$$

We compare our Algorithm 3.1 with Algorithm 1.1, Algorithm 1.2, Algorithm 1 and Algorithm 1. We plot the graphs of errors against the number of iterations in each case with $\|x_n - x_{n-1}\| < 10^{-2}$ as the stopping criterion. The numerical results are reported in Figures 1-4 and Table 1. We choose four different initial values as follows:
Case 1: $x_0 = 88, x_1 = -1$;
Case 2: $x_0 = -75, x_1 = -2$;
Case 3: $x_0 = 86, x_1 = \frac{3}{5}$;
Case 4: $x_0 = 99, x_1 = -0.95$.

TABLE 1. Numerical Results for Example 6.1

	Case 1		Case 2		Case 3		Case 4	
	Iter.	CPU Time	Iter.	CPU Time	Iter.	CPU Time	Iter.	CPU Time
Byrne al. Algo. (1)	7	0.7298	7	0.7108	7	0.7178	7	0.7065
Byrne al. Algo. (2)	8	0.0098	8	0.0066	8	0.0061	8	0.0058
Jailoka al. Algo. (1)	6	0.0086	6	0.0094	6	0.0096	6	0.0108
Jailoka al. Algo. (2)	9	0.0269	9	0.0236	9	0.0254	9	0.0259
Proposed Algo. 3.1	4	0.0068	4	0.0099	4	0.0062	4	0.0069

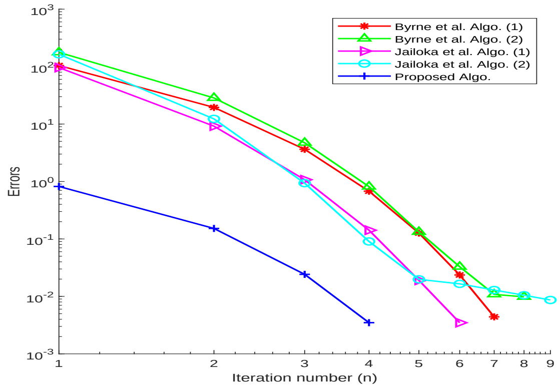

FIGURE 1. Example 6.1 Case 1

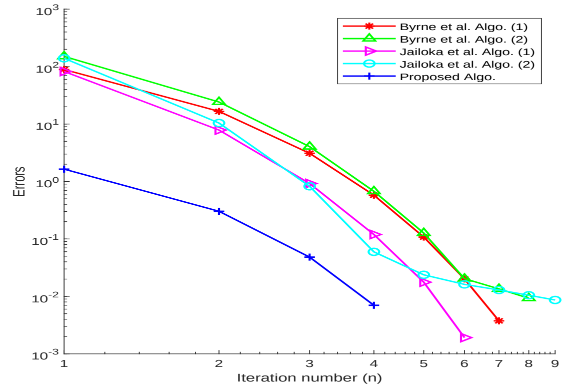

FIGURE 2. Example 6.1 Case 2

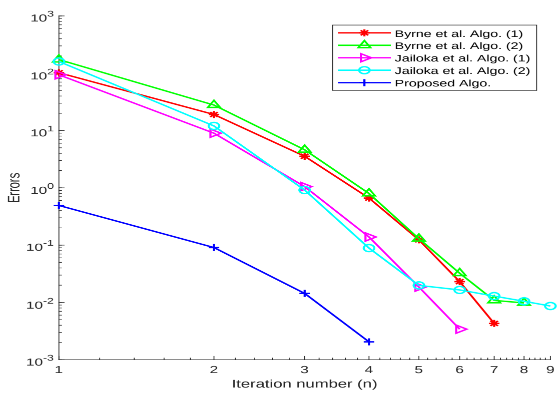

FIGURE 3. Example 6.1 Case 3

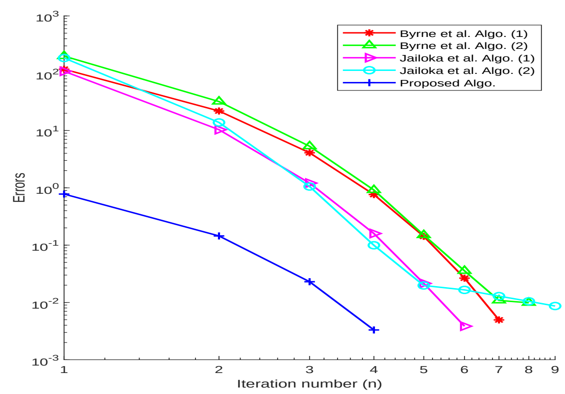

FIGURE 4. Example 6.1 Case 4

Example 6.2. Let $H_1 = H_2 = (l_2(\mathbb{R}), \|\cdot\|_2)$, where $l_2(\mathbb{R}) := \{x = (x_1, x_2, \cdots, x_n, \cdots), x_j \in \mathbb{R} : \sum_{j=1}^{\infty} |x_j|^2 < \infty\}$, $\|x\|_2 = \left(\sum_{j=1}^{\infty} |x_j|^2\right)^{\frac{1}{2}}$ for all $x \in l_2(\mathbb{R})$. Let $A : H_1 \to H_2$ be defined by $Ax = \frac{x}{2}$. Define $B_1 : H_1 \to H_1$ by $B_1 x = \frac{3}{2}x$ and $B_2 : H_2 \to H_2$ by $B_2 x = \frac{5}{2}x$. We take $f(x) = \frac{x}{3}$. We compare our Algorithm 3.1 with Algorithm 1.2, Algorithm 1 and Algorithm 1. We plot the graphs of errors against the number of iterations in each case with $\|x_n - x_{n-1}\| < 10^{-2}$ as the stopping criterion. The numerical results are reported in Figures 5-8 and Table 2.

We choose four different initial values as follows:

Case 1. $x_0 = \left(6, 3, \frac{3}{2}, \cdots\right)$, $x_1 = \left(-\frac{2}{9}, \frac{2}{27}, -\frac{2}{81}, \cdots\right)$;

Case 2. $x_0 = (-8, 4, -2, \cdots)$, $x_1 = \left(2, 1, \frac{1}{2}, \cdots\right)$;

Case 3. $x_0 = \left(-6, \frac{6}{5}, -\frac{6}{25}, \cdots\right)$, $x_1 = \left(1, \frac{1}{2}, \frac{1}{4}, \cdots\right)$;

Case 4. $x_0 = \left(3, \frac{1}{2}, \frac{1}{12}, \cdots\right)$, $x_1 = (1, 0.1, 0.01, \cdots)$.

TABLE 2. Numerical Results for Example 6.2

	Case 1		Case 2		Case 3		Case 4	
	Iter.	CPU Time	Iter.	CPU Time	Iter.	CPU Time	Iter.	CPU Time
Byrne al. Algo. (2)	13	0.0352	13	0.0102	13	0.0117	13	0.0660
Jailoka al. Algo. (1)	11	1.1285	11	0.7158	11	0.7114	11	1.5918
Jailoka al. Algo. (2)	5	0.0256	6	0.0228	5	0.0212	5	0.0309
Proposed Algo. 3.1	3	0.0144	4	0.0087	3	0.0091	3	0.0155

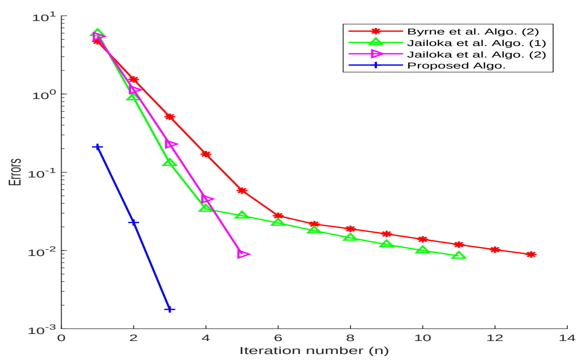

FIGURE 5. Example 6.2 Case 1

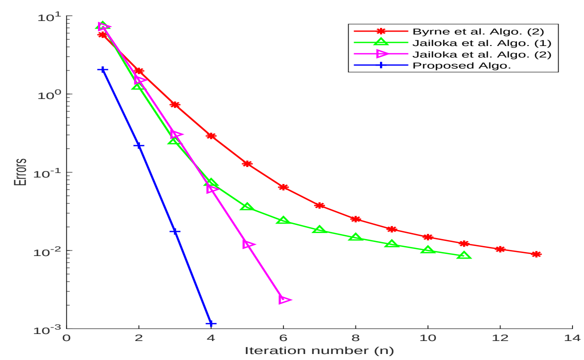

FIGURE 6. Example 6.2 Case 2

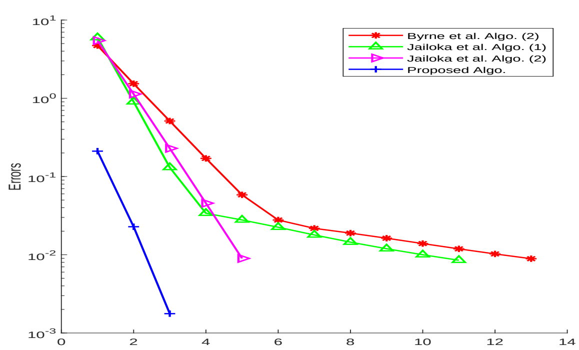

FIGURE 7. Example 6.2 Case 3

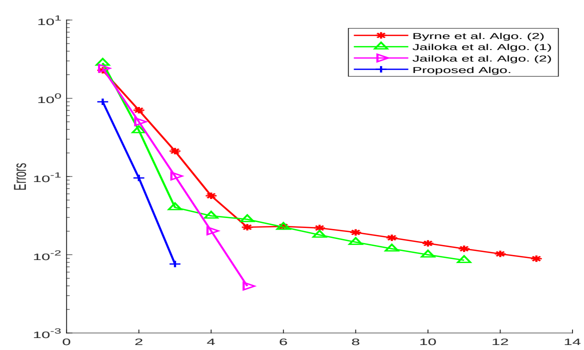

FIGURE 8. Example 6.2 Case 4

7. CONCLUSION

In this paper, we considered the problem of finding common solutions of split null point problems of maximal monotone mappings and fixed point problems of demimetric mappings. We presented a viscosity-type algorithm with inertial factors and obtained strong convergence of the algorithm. Moreover, we applied our result to split minimization problems and split equilibrium problems. Finally, numerical examples were given to illustrate the efficiency of our algorithm.

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