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A STRONG CONVERGENCE THEOREM FOR THE SPLIT COMMON FIXED-POINT PROBLEM OF DEMICONTRACTIVE MAPPINGS

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Abstract. In this paper, we introduce an iterative algorithm to study the split common fixed-point problem of demicontractive mappings in Hilbert spaces. Strong convergence of the proposed algorithm is obtained in Hilbert spaces.
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1. Introduction

Let H_1 and H_2 be two real Hilbert spaces equipped up their own inner product $\langle\cdot,\cdot\rangle$ and norm $\|\cdot\|$. Let $S: H_1 \to H_1$ and $T: H_2 \to H_2$ be two nonlinear mappings. $F(S)$ and $F(T)$ stand for the fixed point sets of S and T, respectively. Let $A: H_1 \to H_2$ be a bounded linear operator with its adjoint A^*.

The split common fixed-point problem (SCFPP) is to find a point $x^* \in H_1$ such that

$$x^* \in F(S) \text{ and } Ax^* \in F(T). \tag{1.1}$$

Specially, if S and T are both orthogonal projections, then SCFPP (1.1) is reduced to the well-known split feasibility problem (SFP) [1], which consists of finding a point x^* such that

$$x^* \in C \text{ and } Ax^* \in Q,$$

where $C \subseteq H_1$ and $Q \subseteq H_2$ are the nonempty closed convex sets and A is a bounded linear operator. These two problems recently have been extensively investigated since they play an import role in various areas including signal processing and image reconstruction (see, e.g., [3, 6, 8, 9, 13, 14, 16, 17] for further details).

To solve the SCFPP (1.1), Censor and Segal [2] proposed the following iterative method: for any initial guess $x_1 \in H_1$, define $\{x_n\}$ recursively by

$$x_{n+1} = S(x_n - \lambda A^*(I-T)Ax_n),$$

where S and T are directed operators. The further generalization of this algorithm was studied by Moudafi [5] for demicontractive operators. Under suitable conditions, he proved that the sequence $\{x_n\}$ converges weakly to a point of the SCFPP (1.1).

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Recently, Wang [11] introduced the following new iterative algorithm for the SCFPP (1.1) of firmly nonexpansive mappings:

Algorithm 1.1. [11] Choose an arbitrary initial guess x_0.

Step 1. Given x_n, compute the next iteration via the formula:

$$x_{n+1} = x_n - \rho_n[x_n - Sx_n + A^*(I-T)Ax_n],\ n \geq 0.$$

Step 2. If the following equality

$$\|x_{n+1} - Sx_{n+1} + A^*(I-T)Ax_{n+1}\| = 0$$

holds, then stop; otherwise go to Step 1.

Under suitable conditions, Wang obtained a weak convergence result.

Very recently, Yao *et al.* [18] extended Wang's [11] result from firmly nonexpansive mappings to more general demicontractive mappings. Also they established a weak convergence theorem. Inspired by the above work, we put forward a question: Can we give a modification of Algorithm 1.1 and get a strong convergence result for the SCFPP (1.1) of demicontractive mappings? The main aim of this paper is to give a positive answer to the above question.

2. Preliminaries

Throughout this paper, let R be the set of real numbers and let H be a real Hilbert space with inner product $\langle \cdot, \cdot \rangle$, and norm $\|\cdot\|$. Let $\{x_n\}$ be a sequence in H. We denote the strong convergence of $\{x_n\}$ to $x \in H$ by $x_n \to x$ and the weak convergence by $x_n \rightharpoonup x$. Let T be a mapping of C into H. We denote by $F(T)$ the set of fixed points of T.

In order to facilitate our investigation in this paper, we recall some definitions as follows.

Definition 2.1. A mapping $T : H \to H$ is said to be

(i) nonexpansive if

$$\|Tx - Ty\| \leq \|x - y\|, \quad \forall\, x, y \in H;$$

(ii) quasi-nonexpansive if $F(T) \neq \emptyset$ and

$$\|Tx - q\| \leq \|x - q\|,\ \forall\,(x,q) \in H \times F(T);$$

(iii) firmly nonexpansive if

$$\|Tx - Ty\|^2 \leq \|x - y\|^2 - \|(I-T)x - (I-T)y\|^2,\ \forall\, x, y \in H;$$

(iv) directed if

$$\|Tx - q\|^2 \leq \|x - q\|^2 - \|x - Tx\|^2,\ \forall\,(x,q) \in H \times F(T);$$

(v) μ-demicontractive if there exists a constant $\mu \in (-\infty, 1)$ such that

$$\|Tx - q\|^2 \leq \|x - q\|^2 + \mu\|x - Tx\|^2,\ \forall\,(x,q) \in H \times F(T),$$

which is equivalent to

$$\langle x - Tx, x - q \rangle \geq \frac{1-\mu}{2}\|x - Tx\|^2. \tag{2.1}$$

Remark 2.1. Notice that 0-demicontractive is exactly quasi-nonexpansive. In particular, we say that it is quasi-strict pseudo-contractive [6] if $0 \leq \mu < 1$. Moreover, if $\mu \leq 0$, every μ-demicontractive mapping becomes a quasi-nonexpansive mapping. Therefore, it is sufficient to only take $\mu \in (0,1)$ in (v) of Definition 2.1, or as the notion of quasi-strict pseudo-contraction due to [6].

Recall that the metric (or nearest point) projection from H onto C is the mapping $P : H \to C$ which assigns to each point $x \in H$ the unique point $P_C x \in C$ satisfying the property $\|x - P_C x\| = \inf_{y \in C} \|x - y\|$. It is well known [10] that $P_C x$ is characterized by the inequality

$$\langle x - P_C x, y - P_C x \rangle \leq 0, \ \ \forall\, y \in C. \tag{2.2}$$

Let us also recall that $I - T$ is said to be demiclosed at zero, if for any sequence $\{x_n\} \subset H$ and $x^* \in H$, we have

$$\left. \begin{array}{l} x_n \rightharpoonup x^* \\ (I-T)x_n \to 0 \end{array} \right\} \Rightarrow x^* = Tx^*.$$

As a special case of the demicloseness principle on uniformly convex Banach spaces given by [4], we know that if C is a nonempty closed convex subset of a Hilbert space H, and $T : C \to H$ is a nonexpansive mapping. Then the mapping $I - T$ is demiclosed on C. Now the following question is naturally raised:

If $T : C \to H$ is quasi-nonexpansive, is $I - T$ still demiclosed on C?

The answer is negative even at 0 as follows.

Example 2.1. [12, Example 2.11] The mapping $T : [0,1] \to [0,1]$ is defined by

$$Tx = \begin{cases} \frac{x}{5}, & x \in [0, \frac{1}{2}], \\ x \sin \pi x, & x \in (\frac{1}{2}, 1]. \end{cases}$$

Then T is a quasi-nonexpansive mapping, but $I - T$ is not demiclosed at 0.

In fact, $F(T) = \{0\}$. For any $x \in [0, \frac{1}{2}]$, we have

$$|Tx - 0| = \left|\frac{x}{5} - 0\right| \leq |x - 0|,$$

and for any $x \in (\frac{1}{2}, 1]$, we have

$$|Tx - 0| = |x \sin \pi x - 0| \leq |x - 0|.$$

Thus T is quasi-nonexpansive. Taking $\{x_n\} \subset (\frac{1}{2}, 1]$ and $x_n \to \frac{1}{2} (n \to \infty)$, we have

$$|(I - T)x_n| = |x_n[1 - \sin \pi x_n]| \to 0 (n \to \infty).$$

But $T\frac{1}{2} = \frac{1}{10} \neq \frac{1}{2}$, i.e., $(I - T)\frac{1}{2} \neq 0$, so $I - T$ is not demiclosed at 0.

In what follows, we give some lemmas which are needed for our main convergence theorem.

Lemma 2.1. [15] *Assume that* $\{a_n\}$ *is a sequence of non-negative real numbers such that*

$$a_{n+1} \leq (1 - \gamma_n)a_n + \gamma_n \delta_n + \varepsilon_n, \ n \geq 0,$$

where $\{\gamma_n\}$ *is a sequence in* $(0,1)$ *and* $\{\delta_n\}$ *is a sequence in* R *such that*

(i) $\sum_{n=0}^{\infty} \gamma_n = \infty$;

(ii) $\sum_{n=0}^{\infty} \varepsilon_n < \infty$;

(iii) $\limsup_{n \to \infty} \delta_n \leq 0$ *or* $\Sigma_{n=1}^{\infty} \gamma_n |\delta_n| < \infty$.

Then $\lim_{n \to \infty} a_n = 0$.

Lemma 2.2. [6] *Assume C is a closed convex subset of a Hilbert space H. Let $T : C \to C$ be a self-mapping of C. If T is a μ-demicontractive mapping (which is also called μ-quasi-strictly-contraction in* [6]*), then the fixed point set $F(T)$ is closed and convex.*

Lemma 2.3. [7] *[The demiclosedness principle of nonexpansive mappings] If $V : H \to H$ is a nonexpansive mapping, then $I - V$ is demiclosed at zero.*

3. Main results

In this section, we always assume that H_1, H_2 are real Hilbert spaces. Let $S : H_1 \to H_1$ and $T : H_2 \to H_2$ be two demicontractive mappings with constants $\beta \in (0,1)$ and $\mu \in (0,1)$, respectively. Let $A : H_1 \to H_2$ be a bounded linear operators with its adjoints A^*.

We use Ω to denote the solution set of problem (1.1), that is,

$$\Omega = \{z : z \in F(S) \text{ and } Az \in F(T)\} = F(S) \cap A^{-1}(F(T)).$$

Throughout, assume $\Omega \neq \emptyset$.

Algorithm 3.1. Step 1. Choose an anchor $u \in H_1$ and initial guess $x_0 \in H_1$ arbitrarily.
Step 2. If

$$\|x_n - Sx_n + A^*(I-T)Ax_n\| = 0,$$

then stop, and x_n is a solution of problem (1.1); otherwise, go on to the next step.
Step 3. Update x_{n+1} via the iteration formula:

$$x_{n+1} = \alpha_n u + (1-\alpha_n)[x_n - \rho_n(x_n - Sx_n + A^*(I-T)Ax_n)],$$

and return to Step 2.

The following lemma can be found in [18].

Lemma 3.1. *$z^\dagger$ solves (1.1) iff $\|z^\dagger - Sz^\dagger + A^*(I-T)Az^\dagger\| = 0$.*

Proof. If $z^\dagger$ solves (1.1), then $z^\dagger = Sz^\dagger$ and $(I-T)Az^\dagger = 0$. It is obvious that

$$\|z^\dagger - Sz^\dagger + A^*(I-T)Az^\dagger\| = 0.$$

To see the converse, we assume that $\|z^\dagger - Sz^\dagger + A^*(I-T)Az^\dagger\| = 0$. For any $z \in \Omega$, we obtain

$$\begin{aligned} 0 &= \|z^\dagger - Sz^\dagger + A^*(I-T)Az^\dagger\|\|z^\dagger - z\| \\ &\geq \langle z^\dagger - Sz^\dagger + A^*(I-T)Az^\dagger, z^\dagger - z\rangle \\ &= \langle z^\dagger - Sz^\dagger, z^\dagger - z\rangle + \langle A^*(I-T)Az^\dagger, z^\dagger - z\rangle \\ &= \langle z^\dagger - Sz^\dagger, z^\dagger - z\rangle + \langle (I-T)Az^\dagger, Az^\dagger - Az\rangle. \end{aligned} \tag{3.1}$$

Since S and T are demicontractive, we deduce from (2.2) that

$$\langle z^\dagger - Sz^\dagger, z^\dagger - z\rangle \geq \frac{1-\beta}{2}\|z^\dagger - Sz^\dagger\|^2, \tag{3.2}$$

and

$$\langle (I-T)Az^\dagger, Az^\dagger - Az\rangle \geq \frac{1-\mu}{2}\|(I-T)Az^\dagger\|^2. \tag{3.3}$$

By (3.1)-(3.3), we get

$$\begin{aligned} 0 &\geq \langle z^\dagger - Sz^\dagger, z^\dagger - z\rangle + \langle (I-T)Az^\dagger, Az^\dagger - Az\rangle \\ &\geq \frac{1-\beta}{2}\|z^\dagger - Sz^\dagger\|^2 + \frac{1-\mu}{2}\|(I-T)Az^\dagger\|^2. \end{aligned} \tag{3.4}$$

Since $\beta, \mu \in (0,1)$, we deduce $z^\dagger \in F(S)$ and $Az^\dagger \in F(T)$ by (3.4). Therefore, $z^\dagger$ solves the problem (1.1). The proof is completed. □

Based Lemma 3.1, we may assume that Algorithm 3.1 generates an infinite sequence $\{x_n\}$, in general since, otherwise, the algorithm terminates in a finite number of iterations and a solution is found.

Lemma 3.2. *Suppose that $\{x_n\}$ is a bounded sequence such that*

$$\lim_{n\to\infty} \|x_n - Sx_n + A^*(I-T)x_n\| = 0.$$

Then $\lim_{n\to\infty} \|x_n - Sx_n\| = 0$ and $\lim_{n\to\infty} \|(I-T)Ax_n\| = 0$.

Proof. Set $y_n = x_n - Sx_n + A^*(I-T)Ax_n$, $z_n = x_n - \rho_n y_n$. For any $z \in \Omega$, we get

$$\langle y_n, x_n - z\rangle = \langle x_n - Sx_n, x_n - z\rangle + \langle (I-T)Ax_n, Ax_n - Az\rangle.$$

Since $z \in F(S)$ and $Az \in F(T)$, $\|y_n\| \to 0$ and $\{x_n\}$ is bounded, we have from (2.1) that

$$\frac{1-\beta}{2}\|x_n - Sx_n\|^2 + \frac{1-\mu}{2}\|(I-T)Ax_n\|^2 \leq \langle y_n, x_n - z\rangle \leq \|y_n\|\|x_n - z\| \to 0.$$

Therefore, we obtain from $\beta, \mu \in (0,1)$ that $\lim_{n\to\infty} \|x_n - Sx_n\| = 0$ and $\lim_{n\to\infty} \|(I-T)Ax_n\| = 0$. The proof is completed. □

Theorem 3.1. *Let the sequences $\{\alpha_n\} \subseteq (0,1)$ and $\{\rho_n\} \subseteq (0, 2\tau)$, where $\tau = \frac{\min\{1-\beta, 1-\mu\}}{4\max\{1, \|A\|^2\}}$. Assume the following conditions are satisfied:*
(a) $I-S$ and $I-T$ are demiclosed at zero;
(b) $\sum_{n=0}^\infty \rho_n^2 < \infty$;
(c) $\lim_{n\to\infty} \alpha_n = 0$ and $\sum_{n=0}^\infty \alpha_n = \infty$;
(d) $\lim_{n\to\infty} \frac{\alpha_n}{\rho_n} = 0$.
Then the sequence $\{x_n\}$ generated by Algorithm 3.1 converges strongly to a solution z of the problem (1.1), where $z = P_\Omega u$.

Proof. By Lemma 2.2, we have that $F(S)$ and $F(T)$ are both closed convex. Since A is bounded linear, $A^{-1}(F(T))$ is closed convex. Hence Ω is closed convex. Put $z = P_\Omega u$ and set $y_n = x_n - Sx_n + A^*(I-T)Ax_n$ and $z_n = x_n - \rho_n y_n$.

First, we show that $\{x_n\}$ is bounded. Indeed, by (2.1), we have

$$\begin{aligned}
\langle y_n, x_n - z\rangle &= \langle x_n - Sx_n + A^*(I-T)Ax_n, x_n - z\rangle \\
&= \langle x_n - Sx_n, x_n - z\rangle + \langle (I-T)Ax_n, Ax_n - Az\rangle \\
&\geq \frac{1-\beta}{2}\|x_n - Sx_n\|^2 + \frac{1-\mu}{2}\|(I-T)Ax_n\|^2 \\
&\geq \frac{1-\beta}{2}\|x_n - Sx_n\|^2 + \frac{1-\mu}{2\|A\|^2}\|A^*(I-T)Ax_n\|^2 \\
&\geq \frac{\min\{1-\beta, 1-\mu\}}{2\max\{1,\|A\|^2\}}(\|x_n - Sx_n\|^2 + \|A^*(I-T)Ax_n\|^2) \\
&\geq \frac{\min\{1-\beta, 1-\mu\}}{4\max\{1,\|A\|^2\}}(\|x_n - Sx_n + A^*(I-T)Ax_n\|)^2, \\
&= \tau\|y_n\|^2, \qquad (3.5)
\end{aligned}$$

where

$$\tau = \frac{\min\{1-\beta, 1-\mu\}}{4\max\{1,\|A\|^2\}}.$$

It follows from (3.5) and $\rho_n \in (0, 2\tau)$ that

$$\begin{aligned}
\|z_n - z\|^2 &= \|x_n - z - \rho_n y_n\|^2 \\
&= \|x_n - z\|^2 - 2\rho_n\langle y_n, x_n - z\rangle + \rho_n^2\|y_n\|^2 \\
&\leq \|x_n - z\|^2 - 2\rho_n\tau\|y_n\|^2 + \rho_n^2\|y_n\|^2 \qquad (3.6) \\
&= \|x_n - z\|^2 - \rho_n(2\tau - \rho_n)\|y_n\|^2 \\
&\leq \|x_n - z\|^2. \qquad (3.7)
\end{aligned}$$

From Algorithm 3.1 and (3.7), we obtain

$$\begin{aligned}
\|x_{n+1} - z\|^2 &= \|\alpha_n(u - z) + (1-\alpha_n)(z_n - z)\|^2 \\
&\leq \alpha_n\|u - z\|^2 + (1-\alpha_n)\|z_n - z\|^2 \\
&\leq \alpha_n\|u - z\|^2 + (1-\alpha_n)\|x_n - z\|^2.
\end{aligned}$$

By mathematical induction, we get

$$\|x_n - z\|^2 \leq \max\{\|u - z\|^2, \|x_0 - z\|^2\}$$

for all $n \geq 0$. Hence $\{x_n\}$ is bounded. From (3.5) and (3.7), $\{y_n\}$ and $\{z_n\}$ are also bounded. By Algorithm 3.1 and (3.6), we have

$$\begin{aligned}
\|x_{n+1} - z\|^2 &= \|(1-\alpha_n)(z_n - z) + \alpha_n(u - z)\|^2 \\
&\leq (1-\alpha_n)\|z_n - z\|^2 + 2\alpha_n\langle u - z, x_{n+1} - z\rangle \\
&\leq (1-\alpha_n)\|x_n - z\|^2 - 2\rho_n\tau(1-\alpha_n)\|y_n\|^2 + \rho_n^2\|y_n\|^2 \\
&\quad + 2\alpha_n\langle u - z, x_{n+1} - z\rangle \\
&\leq (1-\alpha_n)\|x_n - z\|^2 + \alpha_n b_n + \rho_n^2 M, \qquad (3.8)
\end{aligned}$$

where $M = \sup_{n\geq 0}\{\|y_n\|^2\}$ and

$$b_n = 2\langle u-z, x_{n+1}-z\rangle - 2\tau(1-\alpha_n)\frac{\rho_n}{\alpha_n}\|y_n\|^2.$$

Next, we claim that $\limsup_{n\to\infty} b_n \leq 0$. Since $\{b_n\}$ is bounded from above, $\limsup_{n\to\infty} b_n$ is finite. The condition (b) implies that $\rho_n \to 0$. Then

$$\|x_{n+1}-x_n\| = \|\alpha_n(u-x_n)-(1-\alpha_n)\rho_n y_n\| \leq \alpha_n\|u-x_n\| + \rho_n\|y_n\| \to 0$$

due to $\rho_n \to 0$ and $\alpha_n \to 0$. Taking a subsequence $\{x_{n_k}\} \subseteq \{x_n\}$ such that

$$\begin{aligned}\limsup_{n\to\infty} b_n &= \lim_{k\to\infty}[2\langle u-z, x_{n_k+1}-z\rangle - 2\tau(1-\alpha_{n_k})\frac{\rho_{n_k}}{\alpha_{n_k}}\|y_{n_k}\|^2] \\ &= \lim_{k\to\infty}[2\langle u-z, x_{n_k}-z\rangle - 2\tau(1-\alpha_{n_k})\frac{\rho_{n_k}}{\alpha_{n_k}}\|y_{n_k}\|^2]. \qquad (3.9)\end{aligned}$$

Since $\{x_n\}$ is bounded, we may, with no loss of generality, assume that $\{x_{n_k}\}$ is weakly convergent to some point x^*. Thus

$$\lim_{k\to\infty}\langle u-z, x_{n_k}-z\rangle = \langle u-z, x^*-z\rangle. \qquad (3.10)$$

It follows from (3.9), (3.10) and $\alpha_{n_k} \to 0$ that

$$\lim_{k\to\infty}\frac{\rho_{n_k}}{\alpha_{n_k}}\|y_{n_k}\|^2$$

exists. Therefore, we obtain from condition (d) that

$$\|y_{n_k}\|^2 = \frac{\alpha_{n_k}}{\rho_{n_k}}\frac{\rho_{n_k}}{\alpha_{n_k}}\|y_{n_k}\|^2 \to 0,$$

that is,

$$\lim_{k\to\infty}\|y_{n_k}\| = \lim_{k\to\infty}\|x_{n_k}-Sx_{n_k}+A^*(I-T)Ax_{n_k}\| = 0,$$

which, together with Lemma 3.2, implies that

$$\lim_{k\to\infty}\|x_{n_k}-Sx_{n_k}\| = \lim_{k\to\infty}\|(I-T)Ax_{n_k}\| = 0.$$

Therefore by the condition (a) we have $x^* \in F(S)$ and $Ax^* \in F(T)$, i.e., $x^* \in \Omega$. So from (3.9), (3.10) and (2.2) we obtain

$$\limsup_{n\to\infty} b_n \leq \lim_{k\to\infty}\langle u-z, x_{n_k}-z\rangle = \langle u-z, x^*-z\rangle \leq 0.$$

Finally, we show that $\{x_n\}$ converges strongly to $z = P_\Omega u$. The condition (b) implies that $\sum_{n=0}^{\infty}\rho_n^2 M < \infty$. Applying Lemma 2.1 to (3.8) we get from condition (c) that $\|x_n - z\| \to 0$, that is, the sequence $\{x_n\}$ converges strongly to $z = P_\Omega u$. This completes the proof. □

Remark 3.1. Choose $\rho_n = 2\tau\frac{1}{(n+1)^r}, \alpha_n = \frac{1}{(n+1)^s}, \frac{1}{2} < r < s \leq 1$, where

$$\tau = \frac{\min\{1-\beta, 1-\mu\}}{4\max\{1, \|A\|^2\}}.$$

These sequences $\{\rho_n\}$ and $\{\alpha_n\}$ satisfy conditions $(b)-(d)$ in Theorem 3.1.

If S and T are nonexpansive with $F(S) \neq \emptyset$ and $F(T) \neq \emptyset$, then S and T are demicontractive. By using Lemma 2.3, the condition (a) in Theorem 3.1 is satisfied. Using Theorem 3.1, we have the following corollary.

Corollary 3.1. *If S and T are two nonexpansive mappings and $\Omega \neq \emptyset$. The sequences $\{\alpha_n\}$, $\{\rho_n\}$ and the operator A satisfied the conditions in Theorem 3.1. Then the sequence $\{x_n\}$ generated by Algorithm 3.1 converges strongly to a solution z of problem (1.1), where $z = P_\Omega u$.*

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